Article

Yoga of Mathematics

by Aayush Verma

Mathematics in its regular sense, at least at school levels, seems like a fair way of calculating practical probabilities and measuring the required events. Though not entirely true, we are often told that mathematics, and its fashion sets limits for empirical and practical personalities. However, mathematics is one of the oldest sciences, and pinpointing the moment when it transformed into an abstract inquiry is vague to answer. Yet, mathematics, even in its purest form is neither trivial nor dismissive in the search for truth. In fact, in my opinion, it is the grandeur of all. It is that spirit that keeps a mathematician alive and produce the hidden and overlooked truth of life and mathematics alike. It seems an inquiry of abstract symbols may not lead us to the truth that an individual begs for, but it does and this is what I argue by the term Yoga in this article. The word, Yoga, is a very different word to be used in an essay on mathematics and I am aware of only Grothendieck using it. Such words have diverse meanings and unclear connotations and I would prefer to not define it too for that task is too complicated for me to do here. But I am prompted to say that words like yoga, beauty or truth in mathematics already carry a great aplomb with which we describe the words, however, this confidence should not *yet* be the final one.

As an inquirer, one can ask, what is that ultimate question that we must answer before we are done with our lifetime or at least, say before our retirement? But, if there is one, it need not be answered and there is no guarantee if we get it as some special or unique resolution which can conclude one's (to use the word) spiritual journey. Then what do we seek as mathematicians when there could not be a general and singular consensus about the truth. That demands a good and mature thought since it does not have 'one' answer. We seek of different objects and we run against different maps. It is the ignorance which cuts the clay of all unity. Before I go elusive in my words, let me get you through a remarkable piece of mathematics that inspires me. It is the concept of a 'scheme' and we will briefly look at it. A scheme is a locally ringed space with a topology and its elements are the prime ideals of the ring R.

A scheme is a beautiful example of the correspondence between geometry and algebra where on one hand, we have elements in the affine space which corresponds to the spectrum of the ring which is the collection of the prime ideals. Such correspondences and dualities are very important for mathematicians and physicists. A scheme is a modern theory in algebraic

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geometry which has its history in the pens of Weil, Zariski, and Grothendieck.

Now let us understand the importance of *dualities* in mathematics and physics and why a professor of such subjects should, once in their lifetime, cultivate an appreciation of this culture within themselves. A duality means a mirror of which two sides are equivalent against a simple language translation. Physicists need not to look at string theory for such dualities, because there are many early examples like wave-particle duality of light and many examples afterwards. String theory, however, has a wide web of dualities and theorists (and mathematicians too) appreciate it. Holography is also a manifestation of duality which enables one to understand obscure language of gravity theory through conformal field theory, or more formally known as bulk-boundary duality. Theorists working in string theory and quantum gravity are often prompted to understand the dualities concerning the theory in hand and such works really help in developing the understanding. We can name a lot many examples but simple ones would be holography[1] itself, or S-duality/Electric-Magnetic duality and mirror symmetry.

For mathematicians too, a duality is a very important principle and should be a guiding light for many if not all. This principle can be seen in the works of great mathematicians like Grothendieck, Serre, Atiyah, Connes, and Langlands. Because note that dualities help us to see sectors of the theory that is not immediately available to us with its sole premise. I must also add that dualities are often unnoticed of their brilliance and innocence during first few looks hence one should be patient in when bridging the two sectors. Few recent examples of dualities can be found in the Langlands program[2] (like the classic Taniyama–Shimura–Weil conjecture).

Another important Yoga in mathematics is to really ask questions that should be **asked** but also the questions that no one is asking. At some point, mathematics becomes personal to a human and an inherent beauty appears in the investigation that transcends the boundary of the discipline, then no matter how trivial or nonsensical the query would be, a mathematician should trust their conscience and ask it confidently without any shame. In a letter of May, 1982 to his friend R. Brown, Grothendieck writes[3]

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[1] Holography has, by now, many definitions and frameworks but the most common agenda is to find a theory which is 'dual' to some other theory, say sitting on the boundary. A very successful holography is AdS/CFT (a duality between Anti de Sitter/Conformal Field Theory) introduced by Maldacena (arxiv:hep-th/9711200), which also solves the black hole unitarity problem in AdS since the boundary theory (in this case, a CFT) is unitary.

[2] One of my favorite interplay between mathematics and physics, alongside Gauge theory, is the recent developments of Langlands program in theoretical physics, like that of the Geometric Langlands Program (arxiv:hep-th/0604151) and the Relative Langlands Duality (arxiv:2409.04677).

[3] Available at https://webusers.imj-prg.fr/ leila.schneps/grothendieckcircle/Letters/ LettersGrothendieckRBrown.pdf.

"The introduction of the cipher 0 or the group concept was general nonsense, too, and for a thousand years or two mathematics was more or less stagnating, because nobody was around to make such childish steps..."

Indeed, mathematics is a child's game and has to be played in a childish manner. We should and we must grow with our mathematical maturity but that should not sterilize or saturate our adventurous instincts. It should only make mathematics a more systematic discipline while still holding the roots of triviality and naivety. But mathematics also requires 'rigor' and that is very important for a mathematician to 'acquire'. For the rigor in mathematics can not be transmitted and a modest rigor can only be encouraged or born into a mathematician. Is mathematics then subservient to the rigor? No, that is far from true.

I speak with a young mind, and the problem with a young mind is its susceptibility to big dreams. As a mathematician, these dreams serve a big purpose of drive and inspiration. A drive, closer to the duality idea, is about locality in mathematics. That enabled Grothendieck to develop the topos theory. The drive of a mathematician is toward knowledge - pure and perhaps absurd to others - but for which no apology is needed from either side.

The thing about the personal and innate creativity of mathematics is that it must be ultimately expressed through intellectual labor and rigor. Beyond the rigor, there always lies a simple and basic thought. The need is of an intense and enduring engagement with mathematics, or any other discipline, that can serve as a chance to challenge ourselves and renew our sense of identity - something which is truly personal to us.