A Life in Hilbert Space: Interview with BV Rajarama Bhat

Fig: BV Rajarama Bhat Wikipedia

BV Rajarama Bhat is a mathematician at the Indian Statistical Insitute (ISI), Bangalore. Prof. R Bhat is interested in subjects like quantum probability and operator algebra. He is a strong witness to the ISI ecosystem and served in building the edifice that is known as one of the brilliant places to do mathematics.

In this interview, we explore the sides of a mathematician. Prof. R Bhat takes us through his childhood, upbringing, his passions for gardening, and his mathematics. We discuss various things like education in India, Prof. Parthasarathy's impact on his life, operator algebras and the progress in mathematics.

Devang Bajpai: How was your childhood? Were you interested in any particular subject while you were growing up? And did you always want to become a mathematician?

BV Rajarama Bhat: Well, I was born in a small village called Alankar in coastal Karnataka in a poor family. There was nothing special about my childhood. My father was an automobile engineer whereas my mother was a farmer. And yes, even during my school days, I was interested in mathematics.

DB: So how was your schooling and was there any teacher influence on you that you still remember to this day?

RB: Yeah, I studied in a local government school within a kilometer away from home, so we used to walk every day and in those days, even in government schools at least the teaching standards were good. Though the facilities were nothing great, the teaching standards were decent. The main thing which motivated me into mathematics was not any of the teachers but a Kannada Literary magazine called *Kasturi*.[1] They used to call themselves as Kannada's Reader's Digest and there was an author by the name of G.T. Narayana Rao who used to write very nice science and mathematics articles. Though the magazine itself was for the general public, in those articles he would go into real depth in mathematics. He used to put even open problems in mathematics, like the problems about prime numbers and Collatz conjecture and many other things, so that made me interested in mathematics.

I especially remember when I was in school, in Class 11^{th}, there was a question in a textbook, which asked to prove that $2n - 1$ is a prime, iff n is a prime. I looked at it and then I realized that it cannot be correct because I had seen from the articles of Narayana Rao that there is no such algorithm or a simple way of getting a formula of obtaining larger and larger primes. If this statement was correct, one could do that, because one can always take the power by two and subtract one and continue that. So that's what I remember. So it was clearly a wrong question, and these articles of Narayana Rao motivated me into thinking about mathematical problems. There was also another magazine by the name *Bala Vijnana* which means 'Science for Youngsters' that used to have nice articles generally, not just in mathematics, but also about science that got me interested in things like identifying constellations or identifying birds. That is what I think motivated me to mathematics in general.

Aayush Verma: So it is clear that reading was impactful in your early Life.

RB: Yeah, I was sort of a bookworm myself along with my brother. We used to read books all the time and my mother was indeed a big influence. Though we were poor, my mother knew that education was very important in life and so she educated all of us and we had access to books.

AV: That is good to hear. So how did you start your undergraduate and where did you pursue it?

RB: So after Class 12^{th}, I tried for usual institutes like IITs, and NITs and I got selected at NIT Calicut. But my eldest brother, who was studying at IIT Kanpur happened to have some Bengali friends, so he knew about ISI [Indian Statistical Institute] and he told me that if I'm interested in mathematics, maybe ISI is the place. So I came to Bangalore and wrote the entrance test and got selected and then went to ISI Kolkata and did B.Stat from there.

AV: And how was your experience there?

RB: That was, of course, an experience of a lifetime. I came from a small village and had no

[1] https://en.wikipedia.org/wiki/Kasthuri(magazine)

command of English or Hindi. I had studied Hindi as well as English in school but it was not in the English medium. It was in Kannada medium. Most of my friends in Kolkata were from Bengal and so I learned Bengali from them. All the people at ISI were very nice to me. A good thing about ISI in those days was that even though ragging was rampant in most institutions, there was no ragging in ISI Kolkata and so I landed in a very friendly place. It was nice. I learned Bengali and also about Kolkata in general. The teaching there, of course, was of very top quality and one teacher that I owe a lot to is *Prof. B. V. Rao*. He taught us Probability for several semesters and he was the favorite teacher of our entire class and actually, we thought that we were his favorite students, so we were very happy.

AV: What other courses do you remember taking there?

RB: Of course, we had all the basic courses in Analysis, Statistics, and Algebra. We had teachers like A.B. Raha, A.K. Roy, and many others. The B.Stat course has mainly statistics and mathematics. Though we had a little bit of some other things like Geology and Physics but the focus was only Mathematics and Statistics, and Mathematics was taught in a very rigorous way. It gave us a solid foundation for further studies in Mathematics.

AV: After Kolkata, did you think of going to Bangalore for your Master's?

RB: No. Actually, we were in Kolkata for the three years of B.Stat and one year of M.Stat and then some of us decided that we would move to Delhi. So I went to Delhi along with many of my classmates and some of us wanted to do mathematics whereas some were interested in economics. For both of these fields, mathematics as well as economics, ISI Delhi was a good place and so we thought of moving to Delhi. We did M.Stat second year in Delhi, so some of us took the specialization that had the maximum amount of mathematics and some others chose economics. Then we had the privilege to learn from some great teachers like K. R. Parthasarathy, K.B. Sinha, Rajendra Bhatia, and R.L. Karandikar among many others.

AV: Did you start your PhD there?

RB: Yes, after M.Stat's second year there, I could not go abroad. I did apply abroad, but didn't get selected at any of the top places like Princeton, Berkeley, and such places. I was not interested in going to some of the lower-level institutions so I continued with my PhD there at ISI Delhi.

AV: How was ISI Delhi different from your undergraduate days? And do you remember anything particular about ISI during these PhD days?

RB: Yeah, so that was a great time in my life because when you are pursuing undergraduate or master's you are most of the time busy with studying and preparing for exams, but in PhD, you get more free time. So we had a very good time there. Again, I had friends studying mathematics and economics. Something special in Delhi for me was that I got an opportunity to run a small, free library for the kids of ISI staff, such as the children of security staff, drivers, and others. The kids there taught me colloquial Hindi and it was a very nice experience to interact with those small kids. They used to call me 'Raja Bhaiyya'. I used to give them one mathematics puzzle every week and whoever solved it would get a poster. There was a sports magazine that gave a free poster every week and I would distribute that to them. It was a great time interacting with those kids.

Regarding PhD study, we did not get any office space so I used to spend most of my time in the ISI library on the second floor. It was full of bound copies of old volumes of journals. You may have accessed MathSciNet. In those days, Math Reviews came as thick books which gave you a list of all the publications in mathematics reviewed and summarized- as to which paper contains what. I used to browse through all of them to look at whatever I found interesting. It was a great experience and very fruitful too. Unlike MathSciNet, where everything is online and you get what you search for when it is physical, you come across surprising topics and results that you were not aware of. But everything was slow, even the email system came towards the end of my PhD days.

AV: And how did you write your PhD thesis? Was it with LaTeX?

RB: Yes, but my first paper was not written with LaTeX. But when it came to the thesis, we had started using LaTeX. The first paper was written with something called Chiwriter. But then someone told us that LaTeX is more useful and it is flexible, so I learned to use that. But before that, I had to learn how to type. There were many institutes that used to teach you this, so I went to a typing center to learn typing. For the first time, I came to know that the letters are not arranged alphabetically in a type-writer but in a peculiar way [QWERTY] so as to minimize the movement of fingers. Before that, we had no experience of typing, whereas these days everybody knows how to type since they have a mobile and the order of letters remains the same.

AV: How did you get in touch with Professor K. R. Parthasarathy?

RB: When we went to Delhi, he was one of our teachers and he taught us many things, like the basics of Probability Theory, Measure Theory, and then later also the basics of Quantum Probability. Since he was our teacher during M. Stat, I decided to do my PhD under him. Though some people said it could be hard, I took the challenge because I liked his style. He was a great teacher and a great expositor.

AV: And what was he working at that point?

RB: He was developing something called *Quantum Stochastic Calculus*. He had also done his PhD from ISI Kolkata. Initially, he started working on Information Theory and then shifted to probability theory. He came in contact with *Prof A. N. Kolmogorov*, known as the founder of modern probability theory, and he even visited him in Russia. K. R. Parthasarathy wrote

Fig: K. R. Parthasarathy in 2023. By BV Rajarama Bhat

books[2] on Probability Theory and Measures on Groups and they are now considered classics. Subsequently, he changed track and moved into Quantum Probability. He was developing Quantum Stochastic Calculus with another mathematician called Robin Hudson. For classical Stochastic Processes, we have the integration coming from It and there is the famous [Kiyoshi] Itô's formula for stochastic integration. Robin Hudson and K. R. Parthasarathy developed a theory of quantum stochastic integration with their own Quantum Ito formula. It established the field of quantum stochastic calculus. That is what he was working on and I started working under him.

AV: What was your thesis problem and how was your experience working with him in those days?

RB: Many professors who are supervisors, generally give a problem to the student to work on, but that was not the style of Prof. Parthasarathy. Whenever there was some seminar or colloquium talk, or he found some concept interesting, he would suggest us to work on that. There were many such suggestions. Some of the problems would be hard, some might be trivial, and perhaps for some of them, the solutions were already known. It was impossible to work on a hundred different problems, so it became my responsibility to pick the right

[2]Parthasarathy, K. R. (2005). Introduction to probability and measure (Vol. 33). Springer.

problem. After many iterations, I finally started working on something called Quantum Weak Markov Processes, and that became quite fruitful. In classical probability, there are stochastic matrices. They are the matrices whose entries are non-negative and the row sums add up to 1. They give rise to Markov chains and in the same way in the continuous time from stochastic semi-groups one constructs Markov processes. The mathematical construction is based on Kolmogorov's existence theorem. Hudson and Parthasarathy had done a similar thing Boson-Fock space setting for semigroups with bounded generators. Boundedness is a rather stringent technical condition. What about the general case, without these kinds of restrictions? Myself and Parthasarathy developed something called the Weak Markov processes, which could work in full generality. These processes mathematically describe quantum open systems. This was the main work I did during my PhD.

DB: So how has ISI been for you? And how do you balance the dynamics of research and teaching there?

RB: After my PhD in ISI Delhi, I went to Pisa in Italy, the touristic town for my postdoc at the University of Pisa under the mentorship of *Prof. F. Fagnola*. I was there for six to seven months and then I got an offer from the Fields Institute, Canada to do a further postdoc. They were running a special operator algebra program and what I was doing was also connected with operator theory and operator algebras. I was very much interested in learning more about operator algebras and I went to Canada. At that time, the Fields Institute was in Waterloo, Canada.

Initially, the Operator Algebra program was for one year, but then they extended it for one more year and Prof. G. Elliott extended my fellowship also by one year. For the second year, the Institute shifted to Toronto. So I was at the Fields Institute for almost two years, and that was a great time for me. There were many leading researchers in Operator Algebra theory, visiting us every month. There were workshops almost every month and lots of visitors. The list of visitors would read like a 'who is who of the field'.

After the Fields Institute, I came to ISI Bangalore as an Associate Professor. That was in 1996. I have been doing both teaching and research ever since at ISI. This institute has given me my life.

DB: How did you get interested in the quantum setting of these stochastic processes, were you interested in physics too?

RB: Well, I had no training in physics. I learned only mathematics, probability, and statistics in ISI. But Parthasarathy was deeply interested in Physics and applications of the theory he was developing in quantum physics. Whatever I'm doing or have done is pure mathematics. However, some of the results have interpretations in quantum theory and maybe some potential applications. But personally, I have no physics background and I am not involved in experiments or physical applications.

PT: So, what are your current research interests?

RB: I continue to work on Quantum Probability. There are various notions in quantum probability- there is something called completely positive maps in operator algebras and they also arise in quantum information theory. In the quantum setting, messages are communicated from one place to another using these quantum channels. The quantum channels are nothing but 'trace preserving completely positive maps'. My main interest is in understanding various mathematical structures related to completely positive maps. More generally, I have been studying operator theory and operator algebras. In Operator theory, we consider individual operators, whereas operator algebra means collections of operators. You can say- *I live in Hilbert space!* I always work on operators in Hilbert Spaces. So that is the basic setting. In recent years, there has been a lot of interest in quantum computation, quantum information, quantum algorithms, etc. All of them use the setting of mathematics coming from Hilbert Space operators. That is the overall domain of my research.

AV: Since you mentioned operator algebra, I will ask this. So, you know this Von Neumann algebra. They were written to rigorize quantum mechanics as it was and now it has been used by physicists a lot, in recent years we have seen this. They use this to understand how the observables look like in some particular space-time or QFT and how gravity affects them.[3] What do you think is so interesting about Von Neumann algebra or let's say operator algebra? Something that is interesting to both mathematicians and physicists.

RB: This requires some detailed explanation. In classical probability, the basic axiomatic setup was given by Kolmogorov.

To describe random events we start with a set called sample space, which is the set of all possible outcomes. Like when you throw a die, the possible outcomes are numbers from 1 to 6. Then we talk about events, which are collections of outcomes. For instance, the outcome is an even number- refers to an event. Then we assign probabilities to these events. If it is a fair die, the possibility of getting an even number is half. If your die is biased, then it may not be half, it may be some other number. This is the setup of classical probability at the basic level. In the quantum setting, the sample space is replaced by a Hilbert space and events are replaced by subspaces of the Hilbert space instead of subsets of the sample space. Then we assign probabilities to them- this is at the first level. Going back to classical probability, though we start with a probability space which consists of the trio of sample space, events, and assignment of probabilities, subsequently we talk mostly about random variables. Mathematically, they are just measurable functions on the sample space. For example, if we say that by throwing a die when the outcome is a certain number, say 4, I will win seven rupees and for some other number, say when the outcome is 6, I win 10 rupees, and so on. So for each outcome, you assign

[3] For an overwiew of the developments in physics, see Witten's talk at Strings 2026 here. The slides are here.

a value, that is, what you win as a consequence of that outcome. This is the concept of random variables. What we study finally are random variables, and not just the basic measure space or probability space. How do different random variables behave? What is their expectation value? What is the variance, the covariance? Like that, we deal with random variables. Their change in time leads to stochastic processes. Now, a similar thing happens in quantum theory: what corresponds to random variables are observables, which are modeled using self-adjoint operators. We consider the collection of self-adjoint operators and the 'algebra' generated by them. This consists of taking all possible sums and products of self-adjoints and closures under certain topologies. Whether you get only a C^*-algebra or a von Neumann algebra, depends on the topology under consideration. These technicalities may not be very relevant but are crucial in mathematical studies. In quantum theory, we need to consider measurement outcomes or events which correspond to projections. General C^*-algebras may not have enough projections, whereas von Neumann algebras are generated by projections and hence have plenty of projections. This is one of the reasons for their importance in quantum theory. Mathematically, they are very beautiful objects, so mathematicians study them just for their beauty without worrying about applications.

AV: As with the Langlands program among the others, we have seen how important sometimes it is to have such broader visions, especially when you are doing mathematics. So in your opinion, how important is it to learn broader areas, other than your research area?

RB: Yes, because in mathematics, the basic ideas are similar. You study different things but the basic setup remains the same. So we say that mathematics is just one subject. You may divide it into algebra, analysis, probability, and so on but they're all interconnected.

You cannot say that I will just learn analysis and will not look at other subjects. It doesn't work that way, since the same ideas get used at different places. At first look, you may say that number theory and complex analysis are not connected at all. But if you go deeper you understand that a lot of number theory is done using complex analysis as the tool. The probability theory models random experiments, and there is nothing random in natural numbers. But now there is probabilistic number theory as a well-established subject. So, mathematics is just one subject. You may separate out certain branches just because it is impossible to study everything and it is convenient to have broad areas separated. So when you specialize you learn a narrow part of a big ocean but often it is seen that ideas and tools from other areas are pretty useful. And often when there are such unexpected connections it becomes interesting and powerful. So it is very important that at the foundation level one studies all fields and you keep your mind open even at an advanced level.

PT: Yeah, since you have so beautifully described the connections in mathematics. For someone who is pursuing mathematics, how essential would you say it is that collaboration between colleagues is? Also, there's a notion or perhaps a misconception

that mathematics is a lone wolf's business. Moreover, how important is this seminar culture of mathematics?

RB: In mathematics, when you concentrate, you often sit at a desk and keep working, either thinking or doing computations. But it is also important to have discussions with others. The seminars and discussions bring up new ideas. You get to see many facets of mathematics which you have not thought of before. If someone lectures on number theory in our colloquiums, which is not my field, I still sit through with the hope that I might find something interesting or enlightening and maybe there are some tools which I can use elsewhere.

I see the speaker approximating a problem in the continuous setup by discretization, then I think that maybe I can also try something similar for the problem I have been working on for the last few months. So that way, there is some stimulation. Many-a-times you'll see possibilities of new directions of research in your own area by attending seminars. Also, the gist of many of the advanced-level topics can be understood much better by listening rather than by reading research articles. To read and understand a paper, it takes a lot more time. But by listening to a talk you can often grasp the main idea of the paper with little effort. And then you decide whether you should read the paper or not. If you think that you like a particular theorem or maybe some proof looks interesting, you can directly jump into that in the paper. So that way, interactions are very important. These days, more and more papers are written through collaborative work because different areas interact together. People are specialists in different fields and it could be useful to take help from others instead of trying to do it alone.

PT: How do you choose the problems that you work on?

RB: One must choose the problem mostly by personal taste. When you attend seminars or you read papers you come across various mathematical problems, 'open problems' to which nobody knows the answer to. But there are other times you ask yourself questions about how you can develop this subject further in some direction. There are results with this kind of assumption, if I remove some of them what happens? So you ask questions to yourself and you start working on that; sometimes you make progress other times you don't. Sometimes you just put such problems on the back burner. This is something I can think about, but I'm not able to do it right now. So you set it aside and start working on something else. Life goes on. So some problems take years to solve because each time you fail and then you try a different method that also doesn't work, then you try another thing and it continues like that. But finally, during some discussions, you get some new ideas and they work. And then you see that the solution has come and it was not very difficult in the first place.
So it is mostly instinct and hard work rather than a definite algorithm.

PT: How do you deal with the frustration in this process?

RB: Yeah, well sometimes there is frustration. Already when you are a PhD student, it is not uncommon. You see that the supervisor has given you some problems, and you are trying so hard and have made little progress. There is some obstacle or the other and you are not able to overcome it. What makes it more difficult at this stage is that there is a deadline by which you need to finish the thesis. But in due course, you get used to barren patches. You see that this is how life in academia is. So when you are not able to go further in research, you go back to mundane academic duties such as proofreading of previous work, responding to emails, etc. If one is a professor there are other routine things like sending recommendations and participating in various committees and so on. Typically, there is a lot of administrative and academic work one is obliged to do. So you get involved in other things and then you come back again. The frustration is usually short-lived, you work on something else and you give yourself more time because you now know by experience that it happens. There are always problems which you don't know how to solve, but then you work on some other problems. There are a few problems where it is very difficult to estimate in the beginning whether a problem is easy or hard. Some questions look easy but take years to solve, whereas some other problems look hard but then somehow, suddenly you get a solution. So it's quite unpredictable, and once you realize it, there is no frustration, really.

DB: Do you see a difference in the way things are done in the Indian institutes and outside India? And how would you describe the current scenario of mathematics in India?

RB: In many universities abroad, PhDs get finished in three years. But in India, usually there is one or two years of course work and a PhD takes five years or more to complete. Abroad, especially in Europe, it is much easier to participate in international conferences and you get an opportunity to interact with top mathematicians in the field. In India, the mathematical community is much smaller and it is hard to get travel grants to attend conferences abroad. On the positive side- in India, the PhD fellowship is more than enough to cover your monthly expenses. Abroad, especially in the USA, typically PhD students are dependent on tutoring and other activities to cover their living expenses and tuition fees.

In my opinion, the mathematics scenario in India is improving and it is much better than before. Earlier, there were very few mathematicians who had research work at the international level, but now the numbers have increased quite substantially. I think the main reason being the new institutes that have come up like IISERs, NISERs, and several new IITs. Earlier, except for a few university departments, the top-level research was being done only in a handful of institutes like ISI, TIFR, or Matscience [IMSc]. Now it has spread to many institutes. Many researchers who have done PhD in these institutes have moved out to other newer institutes and also many who studied abroad are coming back to newer institutes. Even in older IITs, the mathematics departments are increasing in size and even the quality of research is improving quite significantly. So overall, the situation of mathematics is improving. But we are still not at the top level, and there is a long way to go. It is similar to sports, say in the Olympics, we are doing much better than before, but there is a lot to achieve. Compared to developed countries, we are far behind. There are more mathematicians doing mathematics at the international level but one can say that there are hardly any contenders for the Fields Medal from India. So we are not doing research at that level, but we are headed in the right direction.

But at the same time, there are certain issues popping up for those who want to do PhD in mathematics, i.e., the institutes are slowly getting saturated. So the recent PhDs are finding it hard to get postdoctoral or faculty positions. It is becoming more and more competitive. I guess eventually some of them will have to move to industries or to private colleges to get suitable positions. Considering the size of our population, we need to have more institutions of higher education. Maybe that can be achieved with private participation.

There is a lot of talent that has to be channelized. The access to research papers and other resources has improved quite substantially. Earlier, it was very hard to get copies of research papers. But now, you get most of the new articles through the arXiv and the access is free. For published papers without access, you can always email the author and get a copy of the article. Interacting with international researchers was difficult, but now there are several online resources where you can interact with other researchers quite easily. One can attend online seminars, conferences, etc. Recently we are hearing about the proposed plan called '*One nation, one subscription*'. It is being talked about. If implemented, mathematicians of even smaller institutes and remote places would get access to almost all new research papers. I think that this is a very good proposal.

AV: You have mentioned private investments, but I don't think so if in India there are many like The Simons in the US. I see just Infosys here.

RB: Yeah, we are not going in that direction yet. You see, there is a lot of investment in private engineering colleges and medical colleges. But for pure science, there are very few universities. Our public universities are many in number but most of them are not in good shape. Most of the private universities are also not doing well as far as research is concerned. This is a pity. In my opinion, India has a lot of potential to become a teacher for most of the developing countries. We have a lot of manpower, who can be employed in the teaching profession. The students from India and other developing countries can study in India, instead of going to the United States or Europe provided we are able to make use of this opportunity. I think there is a big scope for private investment in developing the educational and research infrastructure of India. At the same time, the government should keep up its investment. The existing institutes need to be supported. The private investment that we have got so far is very little.

AV: How important is it to take small steps while pursuing theoretical sciences? And many are afraid of making these, because they believe these small steps are very trivial, but you know, sometimes it results in some very good progress in mathematics, and sometimes it is only because nobody else cared about it or thought of them as nonsense.

RB: Yeah, it is important. Actually, research publications are rarely because of single big steps. It is mostly a result of many small steps accumulated over a length of time. It starts with- 'I think I can do this' and then you go further and further. It may not be feasible to publish minor results, as established journals may reject them, but it is always possible to put it on the arXiv or somewhere else where others may take notice. What eventually becomes important or unimportant is very difficult to predict. The research developed in one field may get used in some other field. Sometimes, what is thought as purely theoretical becomes applied. Many results of Ramanujan, which were considered to be of only theoretical interest, are now used in coding, among other things. We are in a digital age and everything is being converted into digits. So whether it's audio, video, etc., everything gets converted into just numbers and then you convert it back; so there is mathematics everywhere. Even if you just use your mobile for GPS, it is still doing such heavy computations, and it uses number theory, Fourier analysis, and so on. So it is difficult to predict what will be useful and what will not be useful, and you just go by your instinct. If you think what you are doing is interesting, just go ahead and publish it.

DB: Have you ever thought that if you were not a mathematician, what other field or career would you have pursued?

RB: That is difficult to say. I am interested in the biodiversity around us, like plants, birds, insects, etc. I am especially interested in plants. However, it is difficult to say whether I would have become a botanist or not because I have never studied biology after class tenth. So I think if I had not gotten admission to ISI, maybe I would have got admission to some engineering college and would have ended up as an engineer. I don't know whether I would have been happy with it or not. I got admission to ISI and became a mathematician and I am happy about it.

PT: So sometimes there's a consensus that mathematics is a young person's game. How do you feel about it? And is there any age or period in the career when it becomes hard to make progress as opposed to being young in the field?

RB: No, I don't agree with that. The thing is that the subject of mathematics is developing so much that it is becoming harder and harder to get into new research at a younger age. A lot of preparation is needed just to cover the basics. So if you compare with, say, biology or other areas, you see that mathematics publications come much later. Even during a PhD, you're supposed to go through a lot of coursework, as modern mathematics is quite abstract and technical. It takes years to reach a level where you can understand newer mathematical problems, let alone solve them. There is no age limit for learning, but once you get older, and if you want to learn something new it becomes harder. Often you don't have the required time and also there could be a lack of motivation. But there are people who change to totally different areas quite late in their research careers. It is perhaps true that younger people are not hindered by past experiences, and are not afraid of trying new ideas, whereas the senior people think that this problem is too hard and there is no point in spending time on it. However, there are instances of senior people solving long-standing conjectures after years of hard work and also there are those who change their research areas totally to come up with interesting new results. So I don't agree with the statement that 'there is an age limit'. It is rather up to you. If you think, I'm too old, I don't want to do this then nobody can help. Yes, moving from one field to another does become harder once you are older. But it is definitely feasible if you are really motivated.

PT: Besides mathematics, do you value any of the subjects or art as much? Let's say, besides mathematics, what would you take up as a hobby or something that you like to do in your free time?

RB: Well, planting is my hobby. I like to learn about plants. I have a small farm where I grow various things, like coconut, banana, mango, cashew, and many other fruits, even exotic fruits. There are fruits by name: egg fruit, butter fruit, peanut butter fruit, blackberry jam fruit, etc. I don't know whether you have heard of them. Many new fruit plants are coming to India in recent years and I like to grow them. So planting is my hobby. I like to plant wherever it is feasible to plant a tree. I'm generally interested in nature, specifically plants, but I also like reading books, mostly Kannada books.

PT: So any specific interest in literature or philosophy, perhaps poetry or something?

RB: No poetry. I'm not really into it. I read Kannada novels and things like that.

PT: We, at Anveshanā, are striving to explore and understand the beautiful connection between scholarship and human thoughts. In your view, how important and integral human thoughts are in the field of mathematics? And how crucial it is to start independent inquiries in our lives?

RB: I think, all of mathematics is just human thought. We try to model things in mathematics, say the space around us, we model it using geometry; the dynamics using calculus; random things using probability theory. Like that, we model various phenomena using mathematical logic. In mathematics, we accept a few things as axioms and then see what statements we can derive. So it is just a systematic way of arriving at conclusions, based on some assumptions. Everything is just based on abstract human thoughts.

One can start independent inquiries, but there have been so many people before us, so many philosophers and one has to study that and understand what they were saying. Right from Buddha, there have been so many philosophers. I think one should understand what they have said before starting an independent inquiry. It wouldn't make sense to ignore the thoughts and achievements of others, in my opinion.

AV: In pursuit of theoretical sciences, and mostly mathematics, we often tend to go far away from the grounds. So do you think that it is the moral duty of a mathematician

Fig: Waterapple plants and peanutbutter fruit plants in the farm of BV R. Bhat. By BV Rajarama Bhat

to deliver knowledge, whatever they have gone through without too much abstraction to the general public or let's say experts of other fields? In other words, is the author responsible for the modesty of their thoughts in their research and making it widely accessible? Also, you have said that it is the digital age, and hence everything can be made accessible but it is also to a certain degree where it is accessible to you but if it's too abstract, it may not be. And for someone who wants to take up a new field, they often fear, for example, algebraic geometry or category theory which are so abstract and difficult people may not know where to begin.

RB: Yeah, of course. It's very important to convey to others what you are doing. So for that, one might give expository lectures or write survey articles. But you should realize that not everyone is good at conveying- some are good at doing research but may not be skilled in conveying their ideas. People are different. There are some people who are extraordinarily good mathematicians but they write papers which are very hard to read, whereas there are others who read these things and explain them and their ideas spread to more people. This has happened many times. The original work by some is almost unreadable but there are others who make an effort to understand them and convey them to others in a comprehensible way.

Fig: With visitors in the farm. By BV Rajarama Bhat

I think that has happened even in Indian philosophy. Some people write commentaries of earlier works written in a very cryptic way, so you need someone else to explain what is being said. Not everybody can convey their thoughts and I don't fault them for that, because they might be very good at discovering new things. But an effort from everyone to convey their work, why it is useful, or what is the beauty of it is very much appreciated.

Here I recall my teacher K. R. Parthasarathy, who was a great expositor. He was very good at explaining things and telling you what are the crucial ideas. But unfortunately, not everybody has that kind of capacity.

PT: Do you have any advice for younger minds who want to make a career in mathematics?

RB: *Well, giving advice is always risky, right?* One suggestion I have is that when you want to get into mathematics, don't think of choosing a research problem in advance. Suppose while in college, you have come across a mathematical problem and you start thinking that okay, this problem in number theory looks interesting and I will work on this. Well, it is not practical. As I said before, mathematics has developed so much that it takes time to reach the research level and you should be open. At Bachelors and Masters, it is too early to decide on a topic or a problem for research. You may decide in a Bachelor's or Master's whether you want to pursue mathematics, physics, or biology, that much you decide. But once you get into mathematics, you should go through the courses and slowly think about whether you want to do analysis, algebra or geometry, etc. It takes time to choose the direction and then you finally branch out. Typically, the research problem is chosen by the PhD supervisor and not the research student. You choose your general area of work and the PhD guide. This is a very crucial step, since good mutual understanding between the student and the guide is very important. The guide chooses the problem for you, or you choose it in consultation with the guide. The guide would have a better idea as to what is doable and what is worth working on. In PhD, the work has to be completed within four or five years. After PhD, you have more freedom and then you can choose what you want to work on. You may take up more challenging problems of your choice or even change your topic of further research at this stage.

In summary, during undergraduate or master's level, you should not be choosing the research topic. What you find interesting at that stage may not remain as your interest later on. After joining the PhD program, you may choose your area, and then you may choose a guide, and then you may let your guide choose the research problem for you. It is quite possible that later on you move away from that and you start working on slightly different things or totally different things. You should get into mathematics only if you are really interested because this is not a money-making enterprise. So if you think that you will enjoy doing research, get into this field, not otherwise.