Anveshanā

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ARTICLES • INTERVIEWS

INTERVIEWS WITH

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Dedicated to those who dream boldly, venture endlessly, and share generously

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Acknowledgments

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[1] @glazedonut9

Anveshanā

Dear Seeker,

We hereby embark upon a beautiful journey of the quest for knowledge- Anveshanā; a resource created to explore the perceptions of the known and the unknown, and bring together ideas, art, and perspectives. Anveshanā, more than just a collection of articles, is rather a blend of insights, creativity, and curiosity of human thoughts.

Our goal is beautifully simple: to spread knowledge far and wide, transcending all boundaries. Although our primary focus is on theoretical physics and mathematics, through Anveshanā, we aim to bring together the richness of human thoughts and ideas from different fields and cultures.

Every piece in Anveshanā is part of a journey to comprehend both the world around and the world within. From exploring the beauty of colors to uncovering the yoga of mathematics, the contributors have worked to combine ideas from different areas in which the articles connect the art of generalization, the mystery of emotions, to the wonderful women of sciences, the recent advancements in Machine Learning and exploring the complexities of the quantum world. At the end of this document, you will also find a curated **Resources Palette** containing few resources, though primarily about physics and mathematics.

As we leave you with the pages ahead, we hope Anveshanā becomes a companion to you, inspiring you to look beyond the surface and venture boldly with us.

With warm regards,
Devang Bajpai
Purnima Tiwari
Aayush Verma

The Particle and the Man: Interview with Michael Peskin

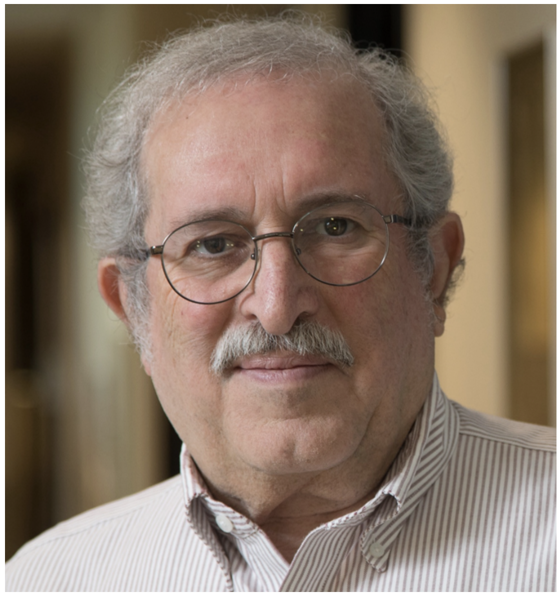

Fig: Michael Peskin SLAC

Michael Peskin is a high-energy physicist at SLAC, Stanford, California. Prof. Peskin is mostly interested in the fundamental interactions of elementary particles. He is the author of the famous textbook 'An Introduction to Quantum Field Theory' with Daniel Schroeder.

Once a PhD student of the Nobel Prize winner Kenneth Wilson, the founder of the Renormalization Program, Prof. Peskin has continued working on the major problems of theoretical and elementary particle physics. In this interview, he recounts his childhood and his life, and we discuss a wide range of subjects from Supersymmetry to quests in a scientific life.

Purnima Tiwari: Good evening, Prof. Peskin. I welcome you on behalf of Anveshanā. We are very glad to have you with us today. So let us begin with your childhood. How was your childhood and did the early formative years shape your interest in any specific field?

Michael Peskin: Well, I had a kind of childhood experience that I think a lot of my contemporary scientists have. We grew up in the suburbs, our parents grew up in the cities. I'm a fourth-generation immigrant from Lithuania or thereabouts. My great-grandparents came over, they worked on a farm. My grandparents owned some small shops in Philadelphia. My parents went to the University of Pennsylvania and became medical doctors, but I wasn't actually interested in becoming a medical doctor. I was interested in more ethereal pursuits and I studied a lot of maths and science and also, poetry and other things. I always thought that I would end up as a scientist. This was a time when the suburban high schools in the US were extremely good, and so a lot of very good scientists who grew up in that era were formed in that way. I always thought that I would become some kind of biological or chemical scientist. In high school, I tried very hard to become a biochemist. And I thought that was the area of science that would really open up as I grew up. Then, I was admitted to Harvard. For two years, I studied chemistry. I learned a lot about quantum mechanics, actually, and physical chemistry. I also learned that I was totally incompetent in the laboratory. So after two years, I decided to change my specialisation to mathematical physics. I started with fluid dynamics and then condensed matter physics. I had this really wonderful advisor named Alan Luther who was guiding me into some of the more advanced topics in condensed matter physics, the behaviour of electrons. At that time, there was a big breakthrough in the theory of phase transitions which was driven by a professor at Cornell named Kenneth Wilson. Wilson actually started out as a high energy physicist; he was a student of Murray Gell-Mann and he worked on problems having to do with many-body physics in nuclear interactions. During the 1960s he developed his own point of view towards quantum field theory, which turned out to be an extremely important breakthrough. And then at the end of the 1960s, he realised that what he had learned was very relevant to condensed matter physics and developed the theory of phase transitions which eventually won him the Nobel Prize.[1] So these developments were happening when I was an undergraduate. Luther suggested that I go to Cornell and work with him as a graduate student. Frankly, it was a fabulous idea, and so I went to Cornell and I was thinking about going into the theory of phase transitions. But just at that time, Wilson decided to go back into particle physics. There were a number of important breakthroughs then. The so-called asymptotic freedom of the theory of the strong interactions (Quantum Chromodynamics or QCD) had just been discovered. He was eager to apply his methods to really understand where strongly interacting particles, hadrons and such, came from. "It's time to solve the strong interactions." After that, there was no turning back. That's how I became a particle physicist.

Aayush Verma: So you didn't decide to do an undergrad in physics and you started with chemistry?

[1] K.G. Wilson, "Renormalization group and critical phenomena. 1. Renormalization group and the Kadanoff scaling picture," Phys. Rev. B 4 (1971), 3174-3183 doi:10.1103/PhysRevB.4.3174.

K.G. Wilson, "Renormalization group and critical phenomena. 2. Phase space cell analysis of critical behavior," Phys. Rev. B 4 (1971), 3184-3205 doi:10.1103/PhysRevB.4.3184.

https://www.nobelprize.org/prizes/physics/1982/wilson/lecture/

MP: Yes, actually, my degree is in a subject called Chemistry and Physics. Harvard gives this degree if you are half and half in both fields. So that was very convenient for me. I've taken a ton of chemistry courses.

AV: What physics courses you started taking there?

MP: Well, I took freshman physics. At Harvard, there were some really advanced freshman courses. There was *Math 55* where you plunge into Abstract Analysis – the abstract maths of Banach spaces and such things – more or less immediately when you walk in the door. There was a similar course in physics. And then there was a similar course in chemistry. So I took the ones in chemistry and maths but I thought taking all three would be too hard. I took a kind of second rank physics course. I can't say I worked on it very hard because I was taking these other really demanding courses. It wasn't until some time later that I caught up with physics. But as I said, in chemistry, I learned a lot of quantum mechanics. I had *Dudley Herschbach* as a professor. The chemists have their own intuitive way of understanding how quantum mechanics works. And I find that a little more congenial than the physicist's way, so I really enjoyed that. And then it turned out that I was really quite well prepared to do physics. (I needed to catch up on mechanics and electrodynamics, which I did on my own, over the summers.) When I went back into physics, what I wanted to do was to take the graduate-level quantum mechanics course. The department advisor, an elderly experimental professor, thought that I was crazy. But at Harvard, if you want to take advanced courses, they let you do what you want, because there are people who are good enough. If you are not smart enough to go right into the advanced courses, it's your problem, not their problem. So in my junior year, I took graduate-level quantum mechanics from *Arthur Jaffe*, someone who's very famous for doing quantum field theory with mathematical rigor. And I had a great experience with that. I really learned a lot. I was hanging on by my fingernails but it got me to where I wanted to go very fast.

AV: Did you start with Mathematical Physics?

MP: Yes, in my junior year. Then in my senior year, I was able to take Quantum Field Theory from *Sidney Coleman*, who's a very famous lecturer on that subject, and that was also a great experience.

PT: Physics often intersects with fields like computer science, chemistry and even philosophy for a fact. How valuable do you think interdisciplinary approaches are in advancing our understanding of the universe?

MP: Well, there are I think two ways that these fields intersect. One of them is through technology. Experiments in physics are extremely difficult and they require very advanced technology of very different kinds, so you really need to know a lot about chemistry, a lot about semiconductors, electrical engineering and computer science, if you are an experimentalis. Please remember though, that I graduated in 1973, so computers were very different then from what they are now. And it wasn't that everyone had a computer in their pocket that could do serious calculations. To be a theorist, you had to know something about computing. You also had to know a lot of maths because a lot of things that would be done now on computers were done analytically on pencil and paper then. I think all these fields have interesting intersections. Now, how these other fields intersect with physics, it depends a little on what you do. If you do condensed matter physics or semiconductors, chemistry is extremely important. I think in my field of particle physics it's much less important. Unless, of course, you're building a detector, which as a theorist is something that I don't do. Every field borrows from every other field. Those people who know how to work across fields often have advantages, but I think my main advantage was having a lot of experience in theoretical condensed matter physics when I went into particle physics because there are strong analogies between those two kinds of physics. At that time there weren't many people who were conversant in both fields. So that's how I got a little advantage.

DB: You went to do PhD at Cornell. Did you have any particular person in your mind to work with before going to Cornell?

MP: Well, as I told you, I wanted to go work with Ken Wilson and that was very interesting. He had a very unique approach to quantum field theory. Since then his viewpoint has become more canonical, but at that time, it was very unusual. And I learned a lot about quantum field theory. He had many unique insights. Working with him was a funny thing, though. I think we never really established a good working relationship. He gave me some problems. I would sweat for a week and try to work out the things he wanted me to work out. And then I'd go and meet him and I'd explain to him what I had done. And he started asking me questions. And the questions would typically have nothing to do with the paper that I was presenting to him. After going away and thinking about it for another day, I realised that those were the questions that I should have been asking given my results. So then, another week and more effort to try to answer the new questions. And again, the same thing happens. He was really on another level. It's maybe a missed opportunity that I didn't pursue that harder but that's the way it worked out.

DB: How exactly did you come in touch with Wilson?

MP: Well, I applied to graduate school. I was by that time a top physics student at Harvard. So in the university admission process, they paid attention to my application. I probably got a very strong letter of recommendation from Alan Luther. So they thought, I was an attractive prospect. And I really did do some good research at Cornell, but mainly it was learning many things, to get from where I began to becoming an expert in both the particle physics and the condensed matter literature. And maybe more learning than actually doing research. But a lot of interesting things were happening in physics at that time. The discovery of asymptotic freedom, semiclassical analysis, instantons, and, the development of QCD. Maybe for you this is all jargon, but it really was a very active period in theoretical particle physics when many

Kenneth G. Wilson (1936-2013) The Nobel Foundation

new ideas were being uncovered. And I really enjoyed just understanding what was the flow of the subject at that time.

DB: Did you work closely with any other faculty too?

MP: Well, I worked a little with John Kogut. Leonard Susskind was a frequent visitor, and I talked to him a lot. The other Cornell faculty were very strong. Kurt Gottfried, Tung-Mow Yan, also David Mermin over on the Condensed Matter side. So I had a great time there. As I said, I didn't really accomplish so much in research, but I learned a lot and it served me well.

AV: Who was on your thesis committee? And do you remember any questions in particular?

MP: It was Wilson, Kurt Gottfried, and Karl Berkelman, one of the experimentalists at the Cornell synchrotron laboratory. Through this lab, I spent a great deal of time interacting with mainly the graduate students, but also some of the faculty, to learn about experimental particle physics. I really enjoyed this. Particle physics experiments are very complicated and it's kind of a black art. But I could get insight by literally crawling around the lab and looking at all the devices and how people used them to measure things. That was a big part of my education. But Karl Berkelman and I never really got along well with each other. I took a particle physics course that he taught but somehow I ended up not doing all the homework. He gave me a project to do which I did in a very theoretical way that he didn't like. He was hoping that I would just find a good paper in the literature and summarise it for him but instead, I made some kind of abstract model of this phenomenon and he thought it was too simple. My committee mainly asked me about the questions in the theory of the pi and K mesons. This material was something that was very relevant to the subject of my thesis[2] but it was more, let's say, phenomenological, whereas my thesis was more mathematical. But I had studied up on that side and was prepared. And they passed me, so I was happy.

AV: You were basically interested in the theoretical aspects, if I'm right?

MP: Certainly I was interested in them, but that isn't what they asked me about on the thesis exam. They thought I had that down pat. So they asked me questions about the experimental consequences. Fortunately, I'd studied this, and knew how to answer them.

AV: And after Cornell, did you have any place in your mind where you wanted to work?

MP: I considered a number of exciting places. I was very interested in SLAC at Stanford. I had met *Sidney Drell*, the leader of the SLAC Theory group, a couple of years earlier at a summer school, and he was a very impressive figure. I was interested in Princeton, and also, actually, I had applied for a fellowship to go to Utrecht and study with *Gerard 't Hooft*. But I got into the Harvard Society of Fellows which is a very prestigious postdoctoral appointment, and that really seemed very attractive. *Steven Weinberg* had just joined Harvard. Sidney Coleman was there. Many very strong theorists were there. It was a great place. Sidney used to say that he was the 'Don Vito Corleone'[3] of particle theory. 'I make them the offers they can't refuse.' And it was certainly true for me. Oh! And just by accident, actually, because I didn't know this in advance… *Edward Witten* was a new postdoc there starting about a year before I got there, and he was a big influence on me and everyone else.

AV: Did you get the chance to talk with Witten?

MP: Yes. Really all the time. Except, he was so fast. We'd have a conversation about something, and a couple of days later, he'd come into my office and it was solved. So I had trouble keeping up with him.

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[2] M.E. Peskin, "CHIRALITY CONSERVATION IN THE LATTICE GAUGE THEORY. 1. THE U(1) PROBLEM AND ITS RESOLUTION," CLNS-395.

M.E. Peskin, "CHIRALITY CONSERVATION IN THE LATTICE GAUGE THEORY. 2. DERIVATION OF LOCAL FIELD EQUATIONS," CLNS-396.

[3] One of the main characters in a novel 'The Godfather' by Mario Puzo.

AV: What questions were keeping him busy?

MP: Well then, he was working on non-perturbative aspects of supersymmetry and thinking about $\mathcal{N} = 4$ Super Yang-Mills theory and its consequences, central charges and formal aspects of supersymmetry. I didn't know much about supersymmetry when I went to Harvard. I had to learn because my officemate was Jim Gates. who was at that time one of the very strong young people working in the theory of supergravity. Warren Siegel, who was his collaborator, was also there. So I would talk to them a lot. But that wasn't the direction that I wanted to go in. What I was trying to do was to understand quark confinement much better and make models of that. So in the end, I didn't end up kind of collaborating with these people. But it was very interesting to follow them and I learned a lot of things from them. Again, the more different things you know, the more you can find a way to put these things together in some combinations that are original. I think that's the way that I've been working most of the time.

AV: After Harvard, you decided to join Stanford, if I'm right?

MP: Oh, no, there were a couple of years in between. First of all, the Society of Fellows appointment was three years, but they let you take one year somewhere else. My wife was studying German literature, so she wanted to go to Germany for a year. So I looked at a map of where she would be studying and started drawing circles. Her university was actually not so far from Paris. So I wrote to some people that I had met through my Cornell connections and I was able to spend a year at CEA Saclay, which is a big national laboratory just outside of Paris. At that time, the theoretical physics group at Saclay was very strong. If you know about this quantum field theory, you might know a famous textbook by Claude Itzykson and Jean-Bernard Zuber, and they were both on the staff then. The great physicist Édouard Brézin was also there. And so there were a number of people that I worked rather closely with. I think in the end, I wrote a paper with Itzykson and Zuber, the title of the paper was "The Roughening of Wilson's Surface".[4] And it was about a problem in lattice gauge theory where there's a phase transition but it is not a phase transition that has to do with quark confinement. It's a kind of epiphenomenon, but quite an interesting one. There's a strong analogy in condensed matter physics, and so we explained it. When I came back to the US, I gave a seminar at Columbia University and for the one time in my life, I met *I. I. Rabi*, the great experimenter and Nobel Prize winner. The title of my seminar was the same thing, the roughening of Wilson's surface, and Rabi, meeting me at coffee before the seminar said, "*Hey, Peskin, how do you polish a Wilson surface?*" So that's my experience meeting Rabi, but I think that the theorists did appreciate my seminar.

AV: I was interested to know what brought you to Stanford. Were there any particular works?

[4]C. Itzykson, M. E. Peskin and J. B. Zuber, "Roughening of Wilson's Surface," Phys. Lett. B 95 (1980), 259-264 doi:10.1016/0370-2693(80)90483-9

MP: After Saclay, I went actually for a postdoc for two years at Cornell, where my wife was finishing up her thesis. Actually, she never did finish her thesis. But we decided it was time to move on. And so I applied for a number of positions and I was offered this position at SLAC that I thought was very attractive. As I said, that place had always been on my list as a really good place to be. And I was a big admirer of Sidney Drell, who was the head of the group. So they offered me a position and I came. That was in 1982, and I'm still here, so I must have had a good time.

DB: Your book with Daniel Schroeder on Quantum Field Theory is very well-known and serves as a great piece of literature. What was the story behind the book?

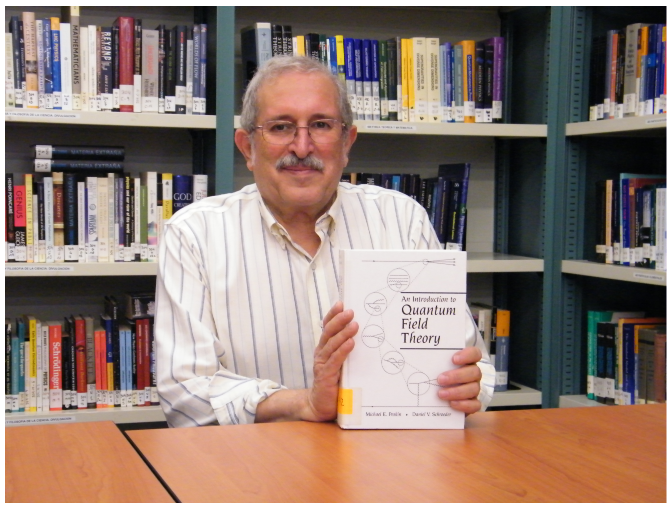

Fig: Michael Peskin in the library of the Institute for Particle Physics/Instituto de Física Corpuscular (IFIC, CSIC-UV) in Valencia Spain holding his classic textbook on Quantum Field Theory. The photo was taken during his visit in September 2016 to deliver a lecture on the Mysteries of the Higgs boson CERN

MP: I'll tell you a little about it. Quantum field theory is a very beautiful subject. I'd studied with the masters, so I really wanted to make an exposition of it. I taught the course at Cornell in those couple years when I was there after my year in France. I taught the course at Stanford, in the 1986 and 1987 academic years. At that time, there was no accepted book on Quantum Field Theory. There was the old book by Sidney Drell and James Bjorken in two volumes, Bjorken and Drell. That was a classic. But, as I said, the 1970s were an important period when people really understood quantum field theory much better and many, many things happened that advanced the field. There was no accepted textbook that was up to date and covered these developments. There was the textbook of Itzykson and Zuber. I tried that for a year at Cornell. My students hated it. It was somehow too French, maybe too dry and rigorous. There was the

Fig: 2018 visit to IISc Bangalore, Peskin having lunch with the students of the high energy physics group. By Michael Peskin

Landau and Lifshitz series book by Berestetskii, Lifshitz, and Pitaevskii. This is a great book with much unique material, but it is idiosyncratic, and my students also found it too difficult. (I'm not sure how Russian students dealt with it, I think by working extremely hard.) But somehow there was no solution for a general graduate textbook on quantum field theory. So I thought that maybe if I had one... I met Dan Schroeder through teaching at Stanford. He was a student in the SLAC theory group but his main interest was becoming a liberal arts physics professor. So I thought this would be a great project for him. I had this big pile of lecture notes, and I gave it to him saying, why don't you just type this up? And we'll be done in a year. In fact, it took us eight years to finish the book. The book came out in 1995, just at the time of the discovery of the top quark, and still, no one had managed to write a book that was a generally accepted modern treatment of quantum field theory. So the book, I must say, was enormously successful. It's still being used, after almost 30 years, and students seem to enjoy it. I travel all over the world and people tell me how much they love the book. It is very pleasing to put together something like that.

DB: Daniel Schroeder also has a book on thermal physics, and he also took that course from you, if I am right?

MP: Indeed, he developed his own approach to this subject. His book is meant for undergraduates. And it is really, I think, the best thermal physics and statistical mechanics book at that level.

AV: You learned Quantum Field Theory from masters like Arthur Jaffe and Sidney Coleman, but when you set out to write, what was the process for you while writing, let's say a book on QFT, where the chances of discouraging a newcomer is very high?

MP: Writing a textbook is very different from doing research. In writing a textbook you are

Fig: 2018 visit to ICTS Bangalore, Peskin giving a talk on topic *"The Search for New Particles at the LHC"*. ICTS

taking ideas that are already well understood by the experts and you're trying to explain them to students. That is a different kind of art. You have to remake the subject so that students can understand it more easily. And I suppose this is something that I happened to be good at. So I thought that if I wrote a book, it had a good chance. And as I say, there was a need in the marketplace for such a textbook, which this one now seems to have filled.

AV: Have you ever been to India? And do you happen to follow any works from the high-energy physics community from India?

MP: I have been to India, but not very often. I've been there three times. I was there for the 2011 international conference on Lepton and Photon Interactions at High Energy, which was held at the Tata Institute of Fundamental Research (TIFR). I gave the summary talk at that meeting. And so that was a very interesting experience. I'd also been there, I think some years before that, I was on a review committee for the Tata Institute and I spent probably three weeks in India between Mumbai and Bangalore. One of my close colleagues, unfortunately now just recently deceased, *Rohini Godbole* was at IISc Bangalore for a long time.

Later, I paid another visit to Rohini in Bangalore. On that trip, I also visited the Tata Institute of Fundamental Research and also the ICTS, which was founded by some of my friends from Tata [TIFR]. So those are the three visits. I never did much tourism in India, it was all about physics. On my first trip, I spent a weekend going to the Ellora and Ajanta caves, but I never, for example, went to the Taj Mahal. *Please excuse me*. Most of these Indian physicists that I know, I'd met in the US. *Spenta Wadia* was a postdoc at SLAC, so I met him in that way. Many of the other leading figures in India were postdocs or spent some time in the United States. Someone that I actually worked with when he was at SLAC was *Ashoke Sen*. Sen was a member of our group at SLAC, just at the explosion of string theory in 1984-85. I don't think we have any joint papers, but we were talking a lot at the time that he was moving

Fig: Rohini Godbole (1952-2024) ICTS

from what he was doing before, which was QCD, into string theory. And so I knew all these people outside of India. It was very congenial when I went to India to visit them.

DB: Can you share more about your interactions and collaborations with Prof. Rohini Godbole?

MP: I met Rohini Godbole at many meetings associated with future accelerators and searches for new particles beyond the Standard Model. In the 1990's, we both became advocates for an electron-positron collider as the next frontier accelerator after the LHC. Through thinking about these questions, we both appreciated the use of measurements of particle polarization to identify new particles and work out their interactions from experimental data. These polarization observables are difficult to measure at hadron colliders, though measurements of the properties of the top quark provide important counterexamples to this statement. But they are very straightforward to measure at e^+e^- colliders (where it is also possible to polarize the initial electron and positron beams), and this provides important and novel information about both Standard and beyond-Standard particles. When I met Rohini, we spent a lot of time discussing these issues. We have only one joint research paper[5] but certainly I learned

[5]R.M. Godbole, M.E. Peskin, S.D. Rindani and R.K. Singh, "Why the angular distribution of the top decay lepton is unchanged by anomalous tbW couplings," Phys. Lett. B 790 (2019), 322-325 doi:10.1016/j.physletb.2019.01.022, arXiv:1809.06285 [hep-ph]

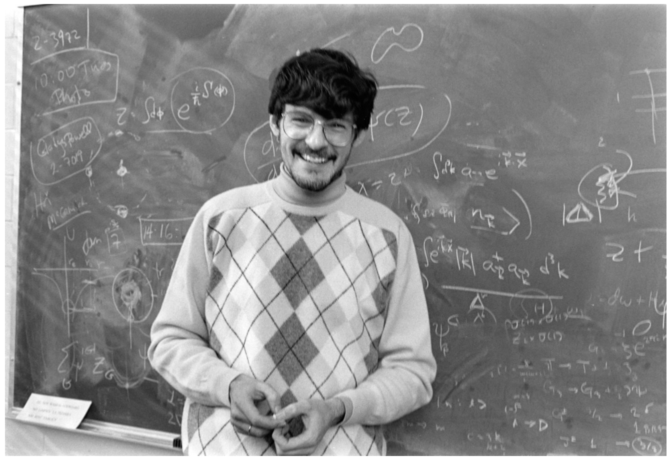

Fig: Emil Martinec in 1991. University of Chicago Photographic Archive

much more from her than this one paper would indicate. She was a gracious host to anyone who passed through IIS Bangalore. Rohini was a rather short woman, but also somewhat wide. I remember being a part of a group, coming home from a conference in Tokyo, that helped her to bring to the airport a pink suitcase that was approximately the same size as she was.

AV: Did Ashoke Sen or anybody try to convince you to do string theory? And what was your take when string theory was just starting?

MP: I had been interested in string theory for a long time. String theory was invented by *Gabriele Veneziano* in 1967 and for a while, it was considered the most interesting approach to the strong interactions. But then it was overtaken by QCD and in the early 70s, it went out of favour. But during the period when it was out of favour, I must say I found it very interesting, because it is an interacting system that you can quantize which generalises quantum field theory. And it's really a very profound theory. So I started studying string theory when I was a graduate student. In 1981, there were papers[6] by the Russian group of *Alexander Polyakov* which gave a new approach to quantizing strings. I got very interested in that, and I worked on this with one of my Cornell colleagues, Orlando Alvarez. Orlando wrote a bunch of papers[7] about that. And I encouraged one of my students, *Emil Martinec*, to work on the

[6] A.M. Polyakov, "Quantum Geometry of Bosonic Strings," Phys. Lett. B 103 (1981), 207-210 doi:10.1016/0370-2693(81)90743-7.
A.M. Polyakov, "Quantum Geometry of Fermionic Strings," Phys. Lett. B 103 (1981), 211-213 doi:10.1016/0370-2693(81)90744-9.

[7] O. Alvarez, "The Static Potential in String Models," Phys. Rev. D 24 (1981), 440 doi:10.1103/PhysRevD.24.440

supersymmetric version[8]. Emil, when he graduated, went to Princeton, and became a member of the Princeton "string quartet" that discovered the heterotic string. So it's a subject I've been interested in for a long time. When string theory came back in 1984-85, I worked hard on it for a few years, but I think I was hoping that there would be very interesting phenomenological applications, given the new understanding. That somehow did not happen, and instead, the subject got very mathematical. And at that point, I dropped out of it and went to work on other things. I'm still a fan of string theory. I think that string theory is likely to be the unique consistent regulator for quantum field theory, but it's going to be a long time before we can really test the stringy predictions of string theory.

DB: Do you believe that intuition plays a significant role in physics? And how do you balance it with the rigorous demand of mathematical proof ?

MP: Everyone has their own level of the relation between intuition and mathematics. There are people who say, 'I have an interesting mathematical problem. I'll do a calculation. Maybe I'll uncover an interesting result and then I can think about the consequences of that.' Other people like to think about the physics, the phenomena or the structures that we use to explain the phenomena, and come up with interesting ideas and then try and find the mathematics to express them properly. For me, it's something in between. I am a believer that these mathematical theories understand more than you do. And so if you do computations, you will uncover things that are surprising, which you can then develop intuition for. I also think that starting from an intuitive basis and trying to find a computation that matches the intuition is a good way to go. And I think in my career, I've done both kinds of things. There are people who are very talented at the purely formal approach. I think the great champion of that probably was Bruno Zumino, the inventor of supersymmetry. There are people who are 'very' intuitive and almost can't do a computation without help, but they know what the answer is. That is very impressive. Leonard Susskind is an example of that point of view. And everyone has to find his place in between.

DB: Well, I believe Feynman has a better combination of both.

MP: You need to realize, though, that these great physicists work extremely hard. We now have the privilege of getting glimpses of Feynman's notebooks. Everything he thought about, Feynman wrote down and seriously considered the consequences of it, and there are very detailed computations in those notebooks. For me, I guess I only do a really detailed computation when I want the numerical answer. I do that much less to try just to understand things. That's a failure of mine, I think.

DB: How has your role at SLAC evolved over the years? Can you share any particularly impactful moments from your work there?

[8] E.J. Martinec, "Superspace Geometry of Superstrings," Phys. Rev. D 28, 2604 (1983) doi:10.1103/PhysRevD.28.2604

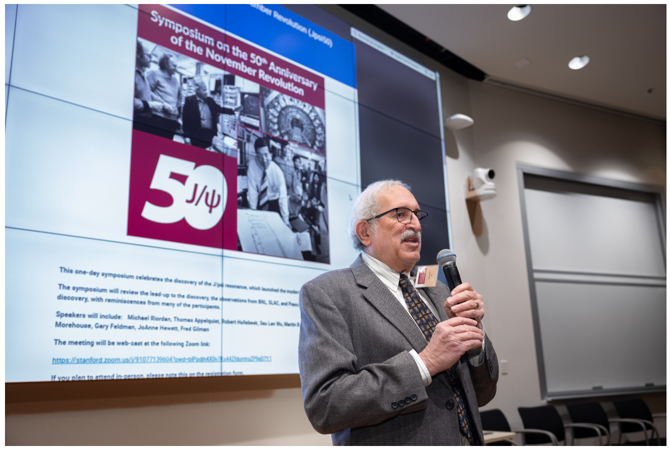

Fig: Peskin at 50th anniversary of the J/ψ discovery held at a week before the interview on Friday. SLAC

MP: The first thing I should say is that I came to SLAC late in the history of SLAC. The really great period for SLAC was in the 1970s. In the late 60s, my future colleagues at SLAC did the experiments called 'deep inelastic scattering', in which they discovered the internal structure of the proton. In 1974, they were doing e^+e^- colliding beam experiments. They discovered the psi particles. That was a tremendous moment when people began to understand that quarks were real. Actually, last Friday, we just celebrated the 50th anniversary of the discovery of the J/ψ particle with a symposium at SLAC. And many of the people who were 30 years old then and are 80 years old now came back to talk about their experiences. It was very moving. At the end of the 1970s, we did experiments on polarised electron deep inelastic scattering at SLAC. Charles Prescott was the leader of that group. He gave the final bit of proof that the weak interaction model of Glashow, Salam, and Weinberg – what we now call the standard model of weak interactions, was correct. And so SLAC was experimentally tremendously influential throughout the 70s. I think after that, much less so. But I came in 1982, so in some sense, I missed the big show. I will never be an old timer at SLAC, in the sense that I was not there during the greatest period. I did get interested in the experiments that people were doing when I arrived, in particular, the electron-positron experiments at the PEP accelerator at SLAC and an accelerator called PETRA (Positron–Electron Tandem Ring Accelerator) at DESY, the electron laboratory in Hamburg, Germany. One very important period for me was the running of a collider called the SLC, the Stanford Linear Collider. On the one hand, to measure the properties of the Z boson, one of the basic quanta of the weak interactions with high precision, and on the other hand, to basically validate the concept of a linear collider for future accelerators. So this is instead of a circular synchrotron, a situation where literally you shoot a beam of electrons and shoot a beam of positrons at it and collide those particles and then in this way, it's possible to reach much higher energies than with the synchrotron.

In 1986, Burton Richter, who was the director of the laboratory, said, 'We'd like to build a linear collider of much higher energy than the SLC, we'd like to understand what the physics is like for that machine.' And so with my colleague David Burke, we organised a study where we simulated events at higher energies, let's say a few hundred GeV, and tried to understand how you would do the experiments and what you would learn. And that's been a large part of my career ever since. One of the things that we learned is that you can produce the Higgs boson very easily, very copiously, and also in a way that it was very easy to recognize. You could measure the properties of the Higgs boson more easily in that setting than in any other that people are talking about. So I've been pushing this idea of studying the Higgs boson and other kinds of exotic particles, at e^+e^- linear colliders since the end of the 1980s. So that's a long time now. Unfortunately, we still haven't built one, but I'm always hopeful.

DB: Well, when did you start working on the Standard Model?

MP: Well, I guess you could say that I have been working on the Standard Model since I was a graduate student. The standard model was new when I was a graduate student. Around the time that the SLC opened, I spent a lot of time thinking about precision weak interactions. And one of the things that I'm known for is the way to use data from these precision experiments on the Z resonance, to try and look for signs of new physics at a higher mass-scale that would slightly perturb the Z. This is something I'm very interested in: How do you use data in a non-trivial way to look for deviations from the Standard Model, and how do you interpret those deviations in terms of possible new physics at higher energies.

DB: So what are your thoughts on the future of the Standard Model? Do you see any particular area that is likely to reshape it or possibly even lead to a new paradigm?

MP: I think that the standard model is obviously incomplete. Let's start with the Higgs boson. In the standard model, all of the particles, including the Higgs boson itself, get mass because the Higgs field is the order parameter of some symmetry breaking. The standard model has a very high degree of symmetry, but the most symmetrical point is unstable. When you go away from that point, the Higgs field gets, what we call an expectation value. It takes a constant value throughout space, and that constant value breaks one of the symmetries of the standard model. This is very analogous to what happens in a magnet or a superconductor. In the magnet, the equations of motion are rotationally symmetric but the spins of iron atoms all line up in the same direction, and so there's a preferred direction, which appears spontaneously. In a superconductor, you start with a normal conductor but then in certain circumstances at low temperatures, the electrons pair up and they provide you a condensed state which can transmute electric currents frictionlessly. This is also described by a kind of symmetry breaking. A long time ago, Yoichiro Nambu proposed[9] that there were such symmetry breakings that

[9]Y. Nambu, "Axial vector current conservation in weak interactions," Phys. Rev. Lett. 4 (1960), 380-382 doi:10.1103/PhysRevLett.4.380

take place in the nuclear forces. In the Standard Model, the whole structure of the model is built on the Higgs field being the agent of such a symmetry breaking. And there are ways to test that this is how particles get masses, for example, you can compare the masses of particles to their couplings to the Higgs boson, which you can measure by measuring the rates of the various Higgs boson decays. And people have done these measurements, to the at the level of 10% or 20%, at the LHC. It's working out extremely well, the Higgs really seems to be the agent of spontaneous symmetry breaking responsible for the masses of all particles. The thing we don't understand is why this happens. The explanation of it in the Standard Model, is simple, but it's totally ad hoc. You put in the values of parameters by hand that you need to get the observed results. And for me, that's just not physics. Somewhere up there should be a new force of nature which interacts with the Higgs boson and forces it to have an unstable potential and condense in a symmetry-breaking way. And for me, this is the most important question in particle physics, maybe even in all of physics. 'What is the new interaction that causes the Higgs to do what it does?' And bound up with that question is the question of the spectrum of quark and lepton masses, the CP violation, many other aspects of the Standard Model are bound up with the behaviour of the Higgs boson. As long as the Higgs boson is just something that we write in our equations without understanding it, we're never going to answer those questions. So there's something out there that we want to find out. I was very much hoping we would get clues to this at the LHC, the CERN Large Hadron Collider. But so far, it seems to be that, though we've discovered the Higgs boson, we haven't discovered clues to its nature beyond the standard model. We have to keep looking for those clues. As I've suggested, maybe you should build an electron-positron collider to measure the Higgs much more precisely. I really feel that the standard model has to break down, because it has missing ingredients, and these must exist in nature. We're physicists, so we have to find them.

DB: Regarding the International Linear Collider (ILC), how do you envision its role in advancing our understanding of fundamental physics? What are your thoughts on its current progress and prospects?

MP: I continue to believe that an e^+e^- linear collider should be the next step after the LHC. Questions about the Higgs boson — why does it have the mass and couplings that it does, why does it obtain a nonzero value throughout space? — are now the most important questions in particle physics. An e^+e^- collider would provide a setting to make very precise measurements of the properties of the Higgs boson, hopefully shedding light on these questions. It would be wonderful to build an accelerator with 10 times the energy of the LHC, but, today, we do not have any workable technology for such an accelerator. But we can learn more about the Higgs boson, and we must.

DB: Since you talked about symmetry breaking, it is a key concept in both particle physics and condensed matter physics, but its application seems to be different in the two fields. Could you explain how symmetry breaking manifests uniquely in

condensed matter physics compared to particle physics?

MP: I don't think it's really different, but in condensed matter physics, it's much easier to understand how spontaneous symmetry breaking works. Condensed matter physics is basically the physics of nuclei and electrons and those are things that we understand very well. So, for example, in a magnet, you can ask, 'Why is it that if you take a block of iron and you lower its temperature, the spins will all line up parallel to one another?' And the answer is that it comes from the atomic physics of iron atoms that are sitting next to each other in the middle that basically the electrons tend to repel. So if you have free electrons, then they like to sit as far from each other as possible. And if you have parallel spins, the Pauli exclusion principle prohibits electrons from coming together, whereas that's not true for opposite spin electrons. So the Pauli exclusion principle and atomic structure, then cause the ground state of a block of iron to have parallel spins. It's not an easy explanation, but it's a very physics-y explanation. Similarly, in a superconductor. *Leon Cooper* discovered the phenomenon of Cooper pairing: At very low temperatures, an electron passing a nucleus can deflect it a little and the nucleus will then have a different force with respect to the next electron that comes by, and that sets up an attractive interaction between the two electrons, which at extremely low temperature can cause them to form a bound state. That bound state is actually a boson, something that does not obey the Pauli exclusion principle, and so these bound states can condense throughout the metal and form a kind of conducting fluid in a metal. Again, it's not an easy explanation. Cooper, John Bardeen, and Robert Schrieffer actually won the Nobel Prize for coming up with this idea.[10] But it's real physics, it is an explanation based on the underlying understanding of the dynamics of electrons. In particle physics, we assume various kinds of symmetry breaking, but again, we don't know why they occur and there's got to be something behind that, I think. So, in some sense, we want to make particle physics more like condensed matter physics by finding the laws that lead to these symmetry breaking expectation values.

AV: What are your current interests and what are you working on nowadays? Moreover, what do you think about the current status of where theoretical high energy physics is and do you happen to follow any work outside of your working zone like black holes, gauge theories, etc?

MP: In the last 10 years or so, I've been pretty well concentrated in my research on the properties of the Higgs boson, experiments to probe the properties of the Higgs boson, and the accelerator physics of future accelerators that might do those experiments. Even more recently, I have been thinking about 10 TeV electron-positron colliders. That is to say, electron-positron colliders that can reach an order of magnitude beyond the energy reach of the LHC. And something that I'm very interested in is this question: How high luminosities can you

[10] L.N. Cooper, "Bound electron pairs in a degenerate Fermi gas," Phys. Rev. 104 (1956), 1189-1190 doi:10.1103/PhysRev.104.1189

get at those colliders? The answer is not so easy to find, because the electrons basically repel each other And electrons and positrons in small bunches strongly interact. So if you imagine such an accelerator, there's a large beam-beam interaction and it's something that actually affects how you design the accelerators, so it's something that I'd like to understand a lot better. More generally, 'What are theories that solve the problem that I asked about? What is the mechanism of the physics understanding of the spontaneous symmetry breaking of the Higgs boson? And how do you make models of that?' So those are the things that I've been working on.

Our group at SLAC is very diverse, and so there are a lot of interesting problems that are outside the area that I discussed. We have, twice a week, seminars where people come in from outside and talk about all these things. So it's great to go to those, I find them really fascinating and I'm trying to learn about these other areas. One of the areas in which we are particularly strong is computational QCD: How do you do calculations in QCD that are at, let's say, at the second, third, or fourth order of perturbation theory? Some of the big experts in that are my colleagues, *Lance Dixon* and *Bernhard Mistlberger*, and they bring in people that they know who work in this area, so I'm learning a lot about it. On the other side, the origin of the dark matter of the universe. I think we have some people in our group, in particular Philip Schuster, Natalia Toro and Rebecca Leane who are very interested in diverse models of the dark matter. They are thinking about how you test these models experimentally, both accelerators and with astrophysical observations. And so, you know, I follow those fields as best I can, although it's not what I'm working on. I think in both of those fields, there's really a lot of progress going on. So maybe we'll see some advances.

AV: What are your thoughts on this year's Nobel Prize in physics that went to John Hopfield and Geoffrey Hinton? Do you follow the works happening in Machine Learning? They are using machine learning in physics, and I believe some of them are using it in experimental high energy physics.

MP: Oh, yes. Actually, some of my close colleagues here at SLAC are interested in machine learning from a variety of points of view. I don't really have any intelligent comments on the Nobel Prize. John Hopfield, of course, is very well known for applying statistical mechanics to a wide variety of systems, including biological systems. It is very beautiful work, and it deserves to be recognized. As far as the computer science aspects, those developments are outside of my area of expertise. Today, AI has taken over many areas of science. One of them is the search for new physics at the LHC. I have a number of colleagues here at SLAC who with amazing ideas on how to use machine learning to better analyse the LHC data and to search for new physics. Personally, I have not played with machine learning tools very much. So I'm probably the wrong person to ask about that.

PT: So in the beginning you had mentioned that you also got into poetry and some philosophy?

MP: Maybe I should just say that I'm not a big fan of philosophy. I studied some philosophy when I was a student, I found it, I don't know... difficult and disappointing in the insights it gave. There's a famous book by *Hans Reichenbach* on the philosophy of quantum mechanics in which he postulates that something can be true, not true, or intermediate — three possibilities — and uses this for his philosophy of quantum mechanics. I always found this to be extremely misleading.

I had the experience when I was a postdoctoral fellow at Saclay of hearing a lecture by the well-known philosopher Bernard d'Espagnat. At the end of the lecture I asked him a question about what quantum spins "really" do. He answered, "You are the kind of person we call a dogmatic materialist." For a week afterward, it was a joke. My co-workers would wave their fingers at me and say, "You dogmatic materialist!". But I feel that, to truly understand quantum mechanics, you must believe that the Schrödinger wavefunction and associated objects such as electron spinors are real. So I am not a fan of philosophical approaches to understanding physics. I think you just have to grapple with the equations and make the best sense of them you can. And, I'm someone who actually believes in the reality of mathematical concepts, in a so-called Platonic point of view that *numbers are real, equations are real*. I believe the Schrödinger wave function is real, at least up to phase. And so this gets me in trouble with philosophers, but I find that it's a very effective way to try to move forward in physics.

PT: Coming back to the magazine, since Anveshanā is striving for a bridge between scholarship and human thought. We would like to know what your interests are, if any, than theoretical physics. Do you have a particular interest in any kind of art? Does it help you to draw inspiration into your work?

MP: I don't know. I've never found...As I've told you, I'm interested in literature. I'm interested in music very much, listening to music, I play the piano very badly but I'm interested in how music works. I see a lot of art when I travel and I enjoy that. I don't find a big relation between that and what I do in physics. It's more of a surface-level connection.

AV: So as a student of physics, or let's say broadly, sciences, one often encounters feelings of discouragement or dissatisfaction with the work and the learning process. So what are your thoughts about this? Do you have, in particular, any advice for young people who want to take a career in physics?

MP: That's a hard question. I think that like all of the sciences, physics requires real devotion. In the US, you know, people talk a lot about what kind of employment do I have after school? Can I get a good job? Can I make a lot of money? None of those things happen if you're very serious about physics. Maybe you can get a comfortable university position, that's about as well as you can do, I think, and it certainly doesn't make up for the hours and hours of work that you have to put in to become a real professional in that subject. So I always tell students that you have to feel that *physics is your calling*. That, the pursuit of science, the pursuit of knowledge is what you imagine your life is going to be. If you're not willing to make that kind of commitment, then you're not going to enjoy hours that you put in doing calculations, building apparatus, trying to do experiments. It has to be that this work gives you joy and brings you closer to what you aspire to. I am sure you appreciate that there is no way that you can do a 50-page calculation and feel joyous the whole time. But you have to understand that it follows a goal that you made for yourself, to improve human knowledge. If you don't feel that, you are not going to succeed. You might as well become a lawyer. So I guess that's the advice that I would give to young people. I think science is very beautiful. I think it's very important that people do it, but it is hard. You have to make a commitment to it.

PT: So we would like to thank you for the time you have given us. And for this beautiful interview that you have given us…

MP: Thank you very much for the opportunity. I hope you found some of this interesting.

Interview

A Life in Hilbert Space: Interview with BV Rajarama Bhat

Fig: BV Rajarama Bhat Wikipedia

BV Rajarama Bhat is a mathematician at the Indian Statistical Insitute (ISI), Bangalore. Prof. R Bhat is interested in subjects like quantum probability and operator algebra. He is a strong witness to the ISI ecosystem and served in building the edifice that is known as one of the brilliant places to do mathematics.

In this interview, we explore the sides of a mathematician. Prof. R Bhat takes us through his childhood, upbringing, his passions for gardening, and his mathematics. We discuss various things like education in India, Prof. Parthasarathy's impact on his life, operator algebras and the progress in mathematics.

Devang Bajpai: How was your childhood? Were you interested in any particular subject while you were growing up? And did you always want to become a mathematician?

BV Rajarama Bhat: Well, I was born in a small village called Alankar in coastal Karnataka in a poor family. There was nothing special about my childhood. My father was an automobile engineer whereas my mother was a farmer. And yes, even during my school days, I was interested in mathematics.

DB: So how was your schooling and was there any teacher influence on you that you still remember to this day?

RB: Yeah, I studied in a local government school within a kilometer away from home, so we used to walk every day and in those days, even in government schools at least the teaching standards were good. Though the facilities were nothing great, the teaching standards were decent. The main thing which motivated me into mathematics was not any of the teachers but a Kannada Literary magazine called *Kasturi*.[1] They used to call themselves as Kannada's Reader's Digest and there was an author by the name of G.T. Narayana Rao who used to write very nice science and mathematics articles. Though the magazine itself was for the general public, in those articles he would go into real depth in mathematics. He used to put even open problems in mathematics, like the problems about prime numbers and Collatz conjecture and many other things, so that made me interested in mathematics.

I especially remember when I was in school, in Class 11^{th}, there was a question in a textbook, which asked to prove that $2n-1$ is a prime, iff n is a prime. I looked at it and then I realized that it cannot be correct because I had seen from the articles of Narayana Rao that there is no such algorithm or a simple way of getting a formula of obtaining larger and larger primes. If this statement was correct, one could do that, because one can always take the power by two and subtract one and continue that. So that's what I remember. So it was clearly a wrong question, and these articles of Narayana Rao motivated me into thinking about mathematical problems. There was also another magazine by the name *Bala Vijnana* which means 'Science for Youngsters' that used to have nice articles generally, not just in mathematics, but also about science that got me interested in things like identifying constellations or identifying birds. That is what I think motivated me to mathematics in general.

Aayush Verma: So it is clear that reading was impactful in your early Life.

RB: Yeah, I was sort of a bookworm myself along with my brother. We used to read books all the time and my mother was indeed a big influence. Though we were poor, my mother knew that education was very important in life and so she educated all of us and we had access to books.

AV: That is good to hear. So how did you start your undergraduate and where did you pursue it?

RB: So after Class 12^{th}, I tried for usual institutes like IITs, and NITs and I got selected at NIT Calicut. But my eldest brother, who was studying at IIT Kanpur happened to have some Bengali friends, so he knew about ISI [Indian Statistical Institute] and he told me that if I'm interested in mathematics, maybe ISI is the place. So I came to Bangalore and wrote the entrance test and got selected and then went to ISI Kolkata and did B.Stat from there.

AV: And how was your experience there?

RB: That was, of course, an experience of a lifetime. I came from a small village and had no

[1] https://en.wikipedia.org/wiki/Kasthuri(magazine)

command of English or Hindi. I had studied Hindi as well as English in school but it was not in the English medium. It was in Kannada medium. Most of my friends in Kolkata were from Bengal and so I learned Bengali from them. All the people at ISI were very nice to me. A good thing about ISI in those days was that even though ragging was rampant in most institutions, there was no ragging in ISI Kolkata and so I landed in a very friendly place. It was nice. I learned Bengali and also about Kolkata in general. The teaching there, of course, was of very top quality and one teacher that I owe a lot to is *Prof. B. V. Rao*. He taught us Probability for several semesters and he was the favorite teacher of our entire class and actually, we thought that we were his favorite students, so we were very happy.

AV: What other courses do you remember taking there?

RB: Of course, we had all the basic courses in Analysis, Statistics, and Algebra. We had teachers like A.B. Raha, A.K. Roy, and many others. The B.Stat course has mainly statistics and mathematics. Though we had a little bit of some other things like Geology and Physics but the focus was only Mathematics and Statistics, and Mathematics was taught in a very rigorous way. It gave us a solid foundation for further studies in Mathematics.

AV: After Kolkata, did you think of going to Bangalore for your Master's?

RB: No. Actually, we were in Kolkata for the three years of B.Stat and one year of M.Stat and then some of us decided that we would move to Delhi. So I went to Delhi along with many of my classmates and some of us wanted to do mathematics whereas some were interested in economics. For both of these fields, mathematics as well as economics, ISI Delhi was a good place and so we thought of moving to Delhi. We did M.Stat second year in Delhi, so some of us took the specialization that had the maximum amount of mathematics and some others chose economics. Then we had the privilege to learn from some great teachers like K. R. Parthasarathy, K.B. Sinha, Rajendra Bhatia, and R.L. Karandikar among many others.

AV: Did you start your PhD there?

RB: Yes, after M.Stat's second year there, I could not go abroad. I did apply abroad, but didn't get selected at any of the top places like Princeton, Berkeley, and such places. I was not interested in going to some of the lower-level institutions so I continued with my PhD there at ISI Delhi.

AV: How was ISI Delhi different from your undergraduate days? And do you remember anything particular about ISI during these PhD days?

RB: Yeah, so that was a great time in my life because when you are pursuing undergraduate or master's you are most of the time busy with studying and preparing for exams, but in PhD, you get more free time. So we had a very good time there. Again, I had friends studying mathematics and economics. Something special in Delhi for me was that I got an opportunity to run a small, free library for the kids of ISI staff, such as the children of security staff, drivers, and others. The kids there taught me colloquial Hindi and it was a very nice experience to interact with those small kids. They used to call me 'Raja Bhaiyya'. I used to give them one mathematics puzzle every week and whoever solved it would get a poster. There was a sports magazine that gave a free poster every week and I would distribute that to them. It was a great time interacting with those kids.

Regarding PhD study, we did not get any office space so I used to spend most of my time in the ISI library on the second floor. It was full of bound copies of old volumes of journals. You may have accessed MathSciNet. In those days, Math Reviews came as thick books which gave you a list of all the publications in mathematics reviewed and summarized- as to which paper contains what. I used to browse through all of them to look at whatever I found interesting. It was a great experience and very fruitful too. Unlike MathSciNet, where everything is online and you get what you search for when it is physical, you come across surprising topics and results that you were not aware of. But everything was slow, even the email system came towards the end of my PhD days.

AV: And how did you write your PhD thesis? Was it with LaTeX?

RB: Yes, but my first paper was not written with LaTeX. But when it came to the thesis, we had started using LaTeX. The first paper was written with something called Chiwriter. But then someone told us that LaTeX is more useful and it is flexible, so I learned to use that. But before that, I had to learn how to type. There were many institutes that used to teach you this, so I went to a typing center to learn typing. For the first time, I came to know that the letters are not arranged alphabetically in a type-writer but in a peculiar way [QWERTY] so as to minimize the movement of fingers. Before that, we had no experience of typing, whereas these days everybody knows how to type since they have a mobile and the order of letters remains the same.

AV: How did you get in touch with Professor K. R. Parthasarathy?

RB: When we went to Delhi, he was one of our teachers and he taught us many things, like the basics of Probability Theory, Measure Theory, and then later also the basics of Quantum Probability. Since he was our teacher during M. Stat, I decided to do my PhD under him. Though some people said it could be hard, I took the challenge because I liked his style. He was a great teacher and a great expositor.

AV: And what was he working at that point?

RB: He was developing something called *Quantum Stochastic Calculus*. He had also done his PhD from ISI Kolkata. Initially, he started working on Information Theory and then shifted to probability theory. He came in contact with *Prof A. N. Kolmogorov*, known as the founder of modern probability theory, and he even visited him in Russia. K. R. Parthasarathy wrote

Fig: K. R. Parthasarathy in 2023. By BV Rajarama Bhat

books[2] on Probability Theory and Measures on Groups and they are now considered classics. Subsequently, he changed track and moved into Quantum Probability. He was developing Quantum Stochastic Calculus with another mathematician called Robin Hudson. For classical Stochastic Processes, we have the integration coming from It and there is the famous [Kiyoshi] Itô's formula for stochastic integration. Robin Hudson and K. R. Parthasarathy developed a theory of quantum stochastic integration with their own Quantum Ito formula. It established the field of quantum stochastic calculus. That is what he was working on and I started working under him.

AV: What was your thesis problem and how was your experience working with him in those days?

RB: Many professors who are supervisors, generally give a problem to the student to work on, but that was not the style of Prof. Parthasarathy. Whenever there was some seminar or colloquium talk, or he found some concept interesting, he would suggest us to work on that. There were many such suggestions. Some of the problems would be hard, some might be trivial, and perhaps for some of them, the solutions were already known. It was impossible to work on a hundred different problems, so it became my responsibility to pick the right

[2] Parthasarathy, K. R. (2005). Introduction to probability and measure (Vol. 33). Springer.

problem. After many iterations, I finally started working on something called Quantum Weak Markov Processes, and that became quite fruitful. In classical probability, there are stochastic matrices. They are the matrices whose entries are non-negative and the row sums add up to 1. They give rise to Markov chains and in the same way in the continuous time from stochastic semi-groups one constructs Markov processes. The mathematical construction is based on Kolmogorov's existence theorem. Hudson and Parthasarathy had done a similar thing Boson-Fock space setting for semigroups with bounded generators. Boundedness is a rather stringent technical condition. What about the general case, without these kinds of restrictions? Myself and Parthasarathy developed something called the Weak Markov processes, which could work in full generality. These processes mathematically describe quantum open systems. This was the main work I did during my PhD.

DB: So how has ISI been for you? And how do you balance the dynamics of research and teaching there?

RB: After my PhD in ISI Delhi, I went to Pisa in Italy, the touristic town for my postdoc at the University of Pisa under the mentorship of *Prof. F. Fagnola*. I was there for six to seven months and then I got an offer from the Fields Institute, Canada to do a further postdoc. They were running a special operator algebra program and what I was doing was also connected with operator theory and operator algebras. I was very much interested in learning more about operator algebras and I went to Canada. At that time, the Fields Institute was in Waterloo, Canada.

Initially, the Operator Algebra program was for one year, but then they extended it for one more year and Prof. G. Elliott extended my fellowship also by one year. For the second year, the Institute shifted to Toronto. So I was at the Fields Institute for almost two years, and that was a great time for me. There were many leading researchers in Operator Algebra theory, visiting us every month. There were workshops almost every month and lots of visitors. The list of visitors would read like a 'who is who of the field'.

After the Fields Institute, I came to ISI Bangalore as an Associate Professor. That was in 1996. I have been doing both teaching and research ever since at ISI. This institute has given me my life.

DB: How did you get interested in the quantum setting of these stochastic processes, were you interested in physics too?

RB: Well, I had no training in physics. I learned only mathematics, probability, and statistics in ISI. But Parthasarathy was deeply interested in Physics and applications of the theory he was developing in quantum physics. Whatever I'm doing or have done is pure mathematics. However, some of the results have interpretations in quantum theory and maybe some potential applications. But personally, I have no physics background and I am not involved in experiments or physical applications.

PT: So, what are your current research interests?

RB: I continue to work on Quantum Probability. There are various notions in quantum probability- there is something called completely positive maps in operator algebras and they also arise in quantum information theory. In the quantum setting, messages are communicated from one place to another using these quantum channels. The quantum channels are nothing but 'trace preserving completely positive maps'. My main interest is in understanding various mathematical structures related to completely positive maps. More generally, I have been studying operator theory and operator algebras. In Operator theory, we consider individual operators, whereas operator algebra means collections of operators. You can say- *I live in Hilbert space!* I always work on operators in Hilbert Spaces. So that is the basic setting. In recent years, there has been a lot of interest in quantum computation, quantum information, quantum algorithms, etc. All of them use the setting of mathematics coming from Hilbert Space operators. That is the overall domain of my research.

AV: Since you mentioned operator algebra, I will ask this. So, you know this Von Neumann algebra. They were written to rigorize quantum mechanics as it was and now it has been used by physicists a lot, in recent years we have seen this. They use this to understand how the observables look like in some particular space-time or QFT and how gravity affects them.[3] What do you think is so interesting about Von Neumann algebra or let's say operator algebra? Something that is interesting to both mathematicians and physicists.

RB: This requires some detailed explanation. In classical probability, the basic axiomatic setup was given by Kolmogorov.

To describe random events we start with a set called sample space, which is the set of all possible outcomes. Like when you throw a die, the possible outcomes are numbers from 1 to 6. Then we talk about events, which are collections of outcomes. For instance, the outcome is an even number- refers to an event. Then we assign probabilities to these events. If it is a fair die, the possibility of getting an even number is half. If your die is biased, then it may not be half, it may be some other number. This is the setup of classical probability at the basic level. In the quantum setting, the sample space is replaced by a Hilbert space and events are replaced by subspaces of the Hilbert space instead of subsets of the sample space. Then we assign probabilities to them- this is at the first level. Going back to classical probability, though we start with a probability space which consists of the trio of sample space, events, and assignment of probabilities, subsequently we talk mostly about random variables. Mathematically, they are just measurable functions on the sample space. For example, if we say that by throwing a die when the outcome is a certain number, say 4, I will win seven rupees and for some other number, say when the outcome is 6, I win 10 rupees, and so on. So for each outcome, you assign

[3]For an overwiew of the developments in physics, see Witten's talk at Strings 2026 here. The slides are here.

a value, that is, what you win as a consequence of that outcome. This is the concept of random variables. What we study finally are random variables, and not just the basic measure space or probability space. How do different random variables behave? What is their expectation value? What is the variance, the covariance? Like that, we deal with random variables. Their change in time leads to stochastic processes. Now, a similar thing happens in quantum theory: what corresponds to random variables are observables, which are modeled using self-adjoint operators. We consider the collection of self-adjoint operators and the 'algebra' generated by them. This consists of taking all possible sums and products of self-adjoints and closures under certain topologies. Whether you get only a C^*-algebra or a von Neumann algebra, depends on the topology under consideration. These technicalities may not be very relevant but are crucial in mathematical studies. In quantum theory, we need to consider measurement outcomes or events which correspond to projections. General C^*-algebras may not have enough projections, whereas von Neumann algebras are generated by projections and hence have plenty of projections. This is one of the reasons for their importance in quantum theory. Mathematically, they are very beautiful objects, so mathematicians study them just for their beauty without worrying about applications.

AV: As with the Langlands program among the others, we have seen how important sometimes it is to have such broader visions, especially when you are doing mathematics. So in your opinion, how important is it to learn broader areas, other than your research area?

RB: Yes, because in mathematics, the basic ideas are similar. You study different things but the basic setup remains the same. So we say that mathematics is just one subject. You may divide it into algebra, analysis, probability, and so on but they're all interconnected.

You cannot say that I will just learn analysis and will not look at other subjects. It doesn't work that way, since the same ideas get used at different places. At first look, you may say that number theory and complex analysis are not connected at all. But if you go deeper you understand that a lot of number theory is done using complex analysis as the tool. The probability theory models random experiments, and there is nothing random in natural numbers. But now there is probabilistic number theory as a well-established subject. So, mathematics is just one subject. You may separate out certain branches just because it is impossible to study everything and it is convenient to have broad areas separated. So when you specialize you learn a narrow part of a big ocean but often it is seen that ideas and tools from other areas are pretty useful. And often when there are such unexpected connections it becomes interesting and powerful. So it is very important that at the foundation level one studies all fields and you keep your mind open even at an advanced level.

PT: Yeah, since you have so beautifully described the connections in mathematics. For someone who is pursuing mathematics, how essential would you say it is that collaboration between colleagues is? Also, there's a notion or perhaps a misconception

that mathematics is a lone wolf's business. Moreover, how important is this seminar culture of mathematics?

RB: In mathematics, when you concentrate, you often sit at a desk and keep working, either thinking or doing computations. But it is also important to have discussions with others. The seminars and discussions bring up new ideas. You get to see many facets of mathematics which you have not thought of before. If someone lectures on number theory in our colloquiums, which is not my field, I still sit through with the hope that I might find something interesting or enlightening and maybe there are some tools which I can use elsewhere.

I see the speaker approximating a problem in the continuous setup by discretization, then I think that maybe I can also try something similar for the problem I have been working on for the last few months. So that way, there is some stimulation. Many-a-times you'll see possibilities of new directions of research in your own area by attending seminars. Also, the gist of many of the advanced-level topics can be understood much better by listening rather than by reading research articles. To read and understand a paper, it takes a lot more time. But by listening to a talk you can often grasp the main idea of the paper with little effort. And then you decide whether you should read the paper or not. If you think that you like a particular theorem or maybe some proof looks interesting, you can directly jump into that in the paper. So that way, interactions are very important. These days, more and more papers are written through collaborative work because different areas interact together. People are specialists in different fields and it could be useful to take help from others instead of trying to do it alone.

PT: How do you choose the problems that you work on?

RB: One must choose the problem mostly by personal taste. When you attend seminars or you read papers you come across various mathematical problems, 'open problems' to which nobody knows the answer to. But there are other times you ask yourself questions about how you can develop this subject further in some direction. There are results with this kind of assumption, if I remove some of them what happens? So you ask questions to yourself and you start working on that; sometimes you make progress other times you don't. Sometimes you just put such problems on the back burner. This is something I can think about, but I'm not able to do it right now. So you set it aside and start working on something else. Life goes on. So some problems take years to solve because each time you fail and then you try a different method that also doesn't work, then you try another thing and it continues like that. But finally, during some discussions, you get some new ideas and they work. And then you see that the solution has come and it was not very difficult in the first place.
So it is mostly instinct and hard work rather than a definite algorithm.

PT: How do you deal with the frustration in this process?

RB: Yeah, well sometimes there is frustration. Already when you are a PhD student, it is not uncommon. You see that the supervisor has given you some problems, and you are trying so hard and have made little progress. There is some obstacle or the other and you are not able to overcome it. What makes it more difficult at this stage is that there is a deadline by which you need to finish the thesis. But in due course, you get used to barren patches. You see that this is how life in academia is. So when you are not able to go further in research, you go back to mundane academic duties such as proofreading of previous work, responding to emails, etc. If one is a professor there are other routine things like sending recommendations and participating in various committees and so on. Typically, there is a lot of administrative and academic work one is obliged to do. So you get involved in other things and then you come back again. The frustration is usually short-lived, you work on something else and you give yourself more time because you now know by experience that it happens. There are always problems which you don't know how to solve, but then you work on some other problems. There are a few problems where it is very difficult to estimate in the beginning whether a problem is easy or hard. Some questions look easy but take years to solve, whereas some other problems look hard but then somehow, suddenly you get a solution. So it's quite unpredictable, and once you realize it, there is no frustration, really.

DB: Do you see a difference in the way things are done in the Indian institutes and outside India? And how would you describe the current scenario of mathematics in India?

RB: In many universities abroad, PhDs get finished in three years. But in India, usually there is one or two years of course work and a PhD takes five years or more to complete. Abroad, especially in Europe, it is much easier to participate in international conferences and you get an opportunity to interact with top mathematicians in the field. In India, the mathematical community is much smaller and it is hard to get travel grants to attend conferences abroad. On the positive side- in India, the PhD fellowship is more than enough to cover your monthly expenses. Abroad, especially in the USA, typically PhD students are dependent on tutoring and other activities to cover their living expenses and tuition fees.

In my opinion, the mathematics scenario in India is improving and it is much better than before. Earlier, there were very few mathematicians who had research work at the international level, but now the numbers have increased quite substantially. I think the main reason being the new institutes that have come up like IISERs, NISERs, and several new IITs. Earlier, except for a few university departments, the top-level research was being done only in a handful of institutes like ISI, TIFR, or Matscience [IMSc]. Now it has spread to many institutes. Many researchers who have done PhD in these institutes have moved out to other newer institutes and also many who studied abroad are coming back to newer institutes. Even in older IITs, the mathematics departments are increasing in size and even the quality of research is improving quite significantly. So overall, the situation of mathematics is improving. But we are still not at the top level, and there is a long way to go. It is similar to sports, say in the Olympics, we are doing much better than before, but there is a lot to achieve. Compared to developed countries, we are far behind. There are more mathematicians doing mathematics at the international level but one can say that there are hardly any contenders for the Fields Medal from India. So we are not doing research at that level, but we are headed in the right direction.

But at the same time, there are certain issues popping up for those who want to do PhD in mathematics, i.e., the institutes are slowly getting saturated. So the recent PhDs are finding it hard to get postdoctoral or faculty positions. It is becoming more and more competitive. I guess eventually some of them will have to move to industries or to private colleges to get suitable positions. Considering the size of our population, we need to have more institutions of higher education. Maybe that can be achieved with private participation.

There is a lot of talent that has to be channelized. The access to research papers and other resources has improved quite substantially. Earlier, it was very hard to get copies of research papers. But now, you get most of the new articles through the arXiv and the access is free. For published papers without access, you can always email the author and get a copy of the article. Interacting with international researchers was difficult, but now there are several online resources where you can interact with other researchers quite easily. One can attend online seminars, conferences, etc. Recently we are hearing about the proposed plan called '*One nation, one subscription*'. It is being talked about. If implemented, mathematicians of even smaller institutes and remote places would get access to almost all new research papers. I think that this is a very good proposal.

AV: You have mentioned private investments, but I don't think so if in India there are many like The Simons in the US. I see just Infosys here.

RB: Yeah, we are not going in that direction yet. You see, there is a lot of investment in private engineering colleges and medical colleges. But for pure science, there are very few universities. Our public universities are many in number but most of them are not in good shape. Most of the private universities are also not doing well as far as research is concerned. This is a pity. In my opinion, India has a lot of potential to become a teacher for most of the developing countries. We have a lot of manpower, who can be employed in the teaching profession. The students from India and other developing countries can study in India, instead of going to the United States or Europe provided we are able to make use of this opportunity. I think there is a big scope for private investment in developing the educational and research infrastructure of India. At the same time, the government should keep up its investment. The existing institutes need to be supported. The private investment that we have got so far is very little.

AV: How important is it to take small steps while pursuing theoretical sciences? And many are afraid of making these, because they believe these small steps are very trivial, but you know, sometimes it results in some very good progress in mathematics, and sometimes it is only because nobody else cared about it or thought of them as nonsense.

RB: Yeah, it is important. Actually, research publications are rarely because of single big steps. It is mostly a result of many small steps accumulated over a length of time. It starts with- 'I think I can do this' and then you go further and further. It may not be feasible to publish minor results, as established journals may reject them, but it is always possible to put it on the arXiv or somewhere else where others may take notice. What eventually becomes important or unimportant is very difficult to predict. The research developed in one field may get used in some other field. Sometimes, what is thought as purely theoretical becomes applied. Many results of Ramanujan, which were considered to be of only theoretical interest, are now used in coding, among other things. We are in a digital age and everything is being converted into digits. So whether it's audio, video, etc., everything gets converted into just numbers and then you convert it back; so there is mathematics everywhere. Even if you just use your mobile for GPS, it is still doing such heavy computations, and it uses number theory, Fourier analysis, and so on. So it is difficult to predict what will be useful and what will not be useful, and you just go by your instinct. If you think what you are doing is interesting, just go ahead and publish it.

DB: Have you ever thought that if you were not a mathematician, what other field or career would you have pursued?

RB: That is difficult to say. I am interested in the biodiversity around us, like plants, birds, insects, etc. I am especially interested in plants. However, it is difficult to say whether I would have become a botanist or not because I have never studied biology after class tenth. So I think if I had not gotten admission to ISI, maybe I would have got admission to some engineering college and would have ended up as an engineer. I don't know whether I would have been happy with it or not. I got admission to ISI and became a mathematician and I am happy about it.

PT: So sometimes there's a consensus that mathematics is a young person's game. How do you feel about it? And is there any age or period in the career when it becomes hard to make progress as opposed to being young in the field?

RB: No, I don't agree with that. The thing is that the subject of mathematics is developing so much that it is becoming harder and harder to get into new research at a younger age. A lot of preparation is needed just to cover the basics. So if you compare with, say, biology or other areas, you see that mathematics publications come much later. Even during a PhD, you're supposed to go through a lot of coursework, as modern mathematics is quite abstract and technical. It takes years to reach a level where you can understand newer mathematical problems, let alone solve them. There is no age limit for learning, but once you get older, and if you want to learn something new it becomes harder. Often you don't have the required time and also there could be a lack of motivation. But there are people who change to totally different areas quite late in their research careers. It is perhaps true that younger people are not hindered by past experiences, and are not afraid of trying new ideas, whereas the senior people think that this problem is too hard and there is no point in spending time on it. However, there are instances of senior people solving long-standing conjectures after years of hard work and also there are those who change their research areas totally to come up with interesting new results. So I don't agree with the statement that 'there is an age limit'. It is rather up to you. If you think, I'm too old, I don't want to do this then nobody can help. Yes, moving from one field to another does become harder once you are older. But it is definitely feasible if you are really motivated.

PT: Besides mathematics, do you value any of the subjects or art as much? Let's say, besides mathematics, what would you take up as a hobby or something that you like to do in your free time?

RB: Well, planting is my hobby. I like to learn about plants. I have a small farm where I grow various things, like coconut, banana, mango, cashew, and many other fruits, even exotic fruits. There are fruits by name: egg fruit, butter fruit, peanut butter fruit, blackberry jam fruit, etc. I don't know whether you have heard of them. Many new fruit plants are coming to India in recent years and I like to grow them. So planting is my hobby. I like to plant wherever it is feasible to plant a tree. I'm generally interested in nature, specifically plants, but I also like reading books, mostly Kannada books.

PT: So any specific interest in literature or philosophy, perhaps poetry or something?

RB: No poetry. I'm not really into it. I read Kannada novels and things like that.

PT: We, at Anveshanā, are striving to explore and understand the beautiful connection between scholarship and human thoughts. In your view, how important and integral human thoughts are in the field of mathematics? And how crucial it is to start independent inquiries in our lives?

RB: I think, all of mathematics is just human thought. We try to model things in mathematics, say the space around us, we model it using geometry; the dynamics using calculus; random things using probability theory. Like that, we model various phenomena using mathematical logic. In mathematics, we accept a few things as axioms and then see what statements we can derive. So it is just a systematic way of arriving at conclusions, based on some assumptions. Everything is just based on abstract human thoughts.

One can start independent inquiries, but there have been so many people before us, so many philosophers and one has to study that and understand what they were saying. Right from Buddha, there have been so many philosophers. I think one should understand what they have said before starting an independent inquiry. It wouldn't make sense to ignore the thoughts and achievements of others, in my opinion.

AV: In pursuit of theoretical sciences, and mostly mathematics, we often tend to go far away from the grounds. So do you think that it is the moral duty of a mathematician

Fig: Waterapple plants and peanutbutter fruit plants in the farm of BV R. Bhat. By BV Rajarama Bhat

to deliver knowledge, whatever they have gone through without too much abstraction to the general public or let's say experts of other fields? In other words, is the author responsible for the modesty of their thoughts in their research and making it widely accessible? Also, you have said that it is the digital age, and hence everything can be made accessible but it is also to a certain degree where it is accessible to you but if it's too abstract, it may not be. And for someone who wants to take up a new field, they often fear, for example, algebraic geometry or category theory which are so abstract and difficult people may not know where to begin.

RB: Yeah, of course. It's very important to convey to others what you are doing. So for that, one might give expository lectures or write survey articles. But you should realize that not everyone is good at conveying- some are good at doing research but may not be skilled in conveying their ideas. People are different. There are some people who are extraordinarily good mathematicians but they write papers which are very hard to read, whereas there are others who read these things and explain them and their ideas spread to more people. This has happened many times. The original work by some is almost unreadable but there are others who make an effort to understand them and convey them to others in a comprehensible way.

Fig: With visitors in the farm. By BV Rajarama Bhat

I think that has happened even in Indian philosophy. Some people write commentaries of earlier works written in a very cryptic way, so you need someone else to explain what is being said. Not everybody can convey their thoughts and I don't fault them for that, because they might be very good at discovering new things. But an effort from everyone to convey their work, why it is useful, or what is the beauty of it is very much appreciated.

Here I recall my teacher K. R. Parthasarathy, who was a great expositor. He was very good at explaining things and telling you what are the crucial ideas. But unfortunately, not everybody has that kind of capacity.

PT: Do you have any advice for younger minds who want to make a career in mathematics?

RB: *Well, giving advice is always risky, right?* One suggestion I have is that when you want to get into mathematics, don't think of choosing a research problem in advance. Suppose while in college, you have come across a mathematical problem and you start thinking that okay, this problem in number theory looks interesting and I will work on this. Well, it is not practical. As I said before, mathematics has developed so much that it takes time to reach the research level and you should be open. At Bachelors and Masters, it is too early to decide on a topic or a problem for research. You may decide in a Bachelor's or Master's whether you want to pursue mathematics, physics, or biology, that much you decide. But once you get into mathematics, you should go through the courses and slowly think about whether you want to do analysis, algebra or geometry, etc. It takes time to choose the direction and then you finally branch out. Typically, the research problem is chosen by the PhD supervisor and not the research student. You choose your general area of work and the PhD guide. This is a very crucial step, since good mutual understanding between the student and the guide is very important. The guide chooses the problem for you, or you choose it in consultation with the guide. The guide would have a better idea as to what is doable and what is worth working on. In PhD, the work has to be completed within four or five years. After PhD, you have more freedom and then you can choose what you want to work on. You may take up more challenging problems of your choice or even change your topic of further research at this stage.

In summary, during undergraduate or master's level, you should not be choosing the research topic. What you find interesting at that stage may not remain as your interest later on. After joining the PhD program, you may choose your area, and then you may choose a guide, and then you may let your guide choose the research problem for you. It is quite possible that later on you move away from that and you start working on slightly different things or totally different things. You should get into mathematics only if you are really interested because this is not a money-making enterprise. So if you think that you will enjoy doing research, get into this field, not otherwise.

Teaching, Learning, and Growing in Mathematics: Interview with Satyanarayana Reddy

Fig: Satynarayana Reddy By S. Reddy

Satyanarayana Reddy is currently working as an Associate Professor at the Department of Mathematics, Shiv Nadar University, India. Prof. Reddy is an active member of the MTTS Trust, funded by NBHM. He did his Ph.D. at Indian Institute of Technology, Kanpur, India. His research fields of expertise include algebraic graph theory, linear algebra, combinatorial matrix theory and Algebraic Number Theory.

Prof. Reddy recounts his early days and influences during the student days. We discuss, among many things, his mathematics, the state of Indian education, MTTS, and his days in Kanpur.

Purnima Tiwari: Good morning Dr Satya. We welcome you on behalf of Anveshanā. So, let us begin with the questions.
How was your childhood? Were you interested in any subject while you were growing up? Did you want to become a mathematician from the start?

Satyanarayana Reddy: No, my childhood was completely enjoyable, rather jolly. I never thought of something as my goal, so mathematics was not at all a part of my life at that time.

In my childhood, I didn't like any particular subject as such. I mostly enjoyed playing games, and roaming around with my friends. Most of my time passed in that way, and there wasn't any particular goal in my mind. Actually, even up until 10+2, there wasn't an inclination towards mathematics.

I have an ordinary family background, so when I came to know that we enter intermediate college after 10th standard in Andhra Pradesh, we would then have to wear pants. Up until then, I used to wear shorts and that was when I got an opportunity to wear trousers and that gave me the motivation to do well in my class 10^{th}. During the 10^{th} class, I started liking mathematics comparatively more than other subjects, but it was mostly natural to me and I scored more, so I started liking it. That was the reason I chose MPC (Mathematics, Physics and Chemistry). But later on, when I went to B.Sc college, I again chose mathematics and computer science because it was the time when computer science was advancing and I thought mathematics, physics, and computer science are good subjects to take up to settle down early. At that time, in particular, in the second year, Group Theory was being taught as a part of our syllabus. That was abstract, up until then everything was normal and I didn't have much motivation to do analysis or geometry. I was not reading things properly until then. Luckily, group theory was a new concept for me and I was enjoying it. I even started explaining it to my friends and then eventually we were introduced to Ring Theory and Vector Spaces and slowly I started getting interested in Algebra. This is what prompted me to pursue mathematics and then I applied for MSc. But most of my friends then were inclined towards computer science since that was believed to provide us with a job. I also applied for MCA and I had my exam at NIT Surathkal but I did not know how to reach there and it took me more time than usual to reach and in the process, I even got sick and had a headache on the exam day and I did not qualify the exam by half a mark. But luckily, simultaneously I applied for an MSc Mathematics at Andhra University and I got selected there, I ranked 20th in the list and I joined as a student in Pure Mathematics.

PT: During your undergrad, were there any other interests? Were you the kind to follow what is taught, or would you also venture outside and learn on your own?

SR: In particular, the Group Theory part had helped me. Somehow I got really interested in the abstractness of the subject. There was a textbook by *S.Chand* that helped me in understanding the concepts. I didn't do anything other than that. I didn't take any tuition at any point in time for mathematics but because of my financial constraints, I had started teaching by then. I taught school children and they would usually come for mathematics and while teaching them I got really interested in the subject. Going through class 9^{th} and $10th$ problems again helped me a lot.

Aayush Verma: Was there any mathematician whose particular style of mathematics and thinking impressed or influenced you?

SR: Frankly speaking, back then, it wasn't really like that. Actually, I didn't get through the course work properly or even read things properly, so completing the syllabus and scoring good marks was my focus then. However, now I realise that the way of reading wasn't correct. There were a few faculties who influenced me by the way they taught but there weren't many people, whom I was inspired by.

AV: Do you remember the names of the faculties who inspired you?

SR: Yes, a few. Prof *K. L. N. Swamy Sir*, in MSc mathematics. At the PhD level, my supervisors influenced me. I had two supervisors, one was *Prof Shashank K Mehta* from the CS department and the other was from the Mathematics department, Prof *A.K. Lal* [Arbind Kumar Lal], and both were very helpful. AK Lal Sir's influence was more. I used to go to his office several times and he would ask me to sit along with him and then would explain things to me and also Shashank K Mehta would allocate one hour to me, every week, in order to read and present so these two people had a good impact on me. But mainly, at the completion of my PhD, in the last semester, there was an MTTS programme conducted in IITK itself and I requested Prof Santanam to attend the MTTS program as an observer. In the first class, I listened to Prof Kumaresan Sir, and I realised that his way of reading was completely different. His teaching style influenced me a lot and afterward, I saw a change in my life. I can say that his one talk completely changed my life. He started with analysis, and up until that point of time, I was completely scared of analysis; even my PhD was in Algebraic Graph Theory. I kept avoiding analysis because I was afraid of it but then I listened to him for the first time and actually, he was just teaching undergrad-level analysis, but I was afraid of even that. He started with the Archimedean property and its applications and proved several results in one class, which really was eye-opening for me. From that point in time, I was more confident.

Also, indirectly, *Keith Conrad* who is an algebraist also helped me. His website articles[1] on Group Theory and Ring Theory helped me a lot. I used to visit his website and he had a good influence on me, though I have never met him but he has helped me. Another person along the same lines was Prof B Sury; his articles helped me a lot. But all of this happened after MTTS came into my life.

AV: You went to do a PhD at the Indian Institute of Technology in Kanpur, how did you make this decision? Was there something or someone you wanted to work with at IITK?

SR: Yeah, actually because of my financial constraints, after my MSc, I started doing jobs to make a living. I was initially working in Andhra Pradesh but after my marriage, I shifted to

[1] https://kconrad.math.uconn.edu/blurbs/

Fig: Campus clicks of Indian Institute of Technology, Kanpur (IITK). By Aayush Verma

Hyderabad and I joined an engineering college there, where I worked for almost five and a half years but by that point in time, there was a QIP (Quality Improvement Programme), which is for teachers, and was held in IITK. So there was a professor of Mechanical Engineering- Prof Bhaskar Das Gupta, and he was the organizer as well as the main instructor. So I came to IITK to attend the program and I was very much impressed by the overall atmosphere there; the peacocks, the beautiful campus, among other things, and I immediately thought of shifting here. Then I went back to my college and talked to my colleagues and I wrote the GATE exam which I wasn't aware of earlier, but luckily my colleagues told me, and one can write JRF too but by then, I had already crossed the age limit, and the only possibility was GATE so I sat for that, and I got a good score. I came to IITK in December, and then in June, I applied for there, but I didn't get the admission then. I then met with Prof Arvind Kumar Lal before leaving IITK, and told him that I have been teaching discrete mathematics for a long period, in particular I was interested in working in Graph Theory so I wanted to work with him. He told me that I had to apply again and qualify to make that possible and he told me that if I cleared the exam and then the interview, we could discuss it. So I started reading again and applied next December and got selected. That is how I got into the PhD program there.

Devang Bajpai: Can you describe your IITK days, did you enjoy the big campus?

SR: Yes, I joined in December 2006. So I was basically there from 2007 onwards. It really was one of the best times in my life, because that was the place where I realised what I knew and what I didn't. The first thing was to unlearn, which was very important for me, and the other was the seminars. It was great for me to be able to listen to those seminars and it was a new experience, and that gave me a little confidence. Another advantage was the different departments, the faculties from very reputed institutes, and the liberty to attend their classes. One can attend any class and that was an advantage in IITK which helped me in taking some courses in CS, and I noticed the importance of Mathematics in all the departments.

Bhaskar Das Gupta influenced me a lot, he is actually from mechanical engineering but he wrote a book on mathematical methods, and later he also conducted the QIP programme again, when I was there as a PhD scholar and he introduced me as a tutor. I was fortunate that he was so cooperative with me throughout my period there. In other aspects too, IITK is a big campus, with a lot of greenery as well as sports facilities. I used to play badminton, go swimming and cycling. Regarding food too, it was the best. The halls, the canteens, and all the facilities are good and available at a nominal cost. In all aspects, IITK was wonderful. Even the Doa canteen, Chemical Canteen, everything is good there. Even the workers there are very polite and respectful. One of the beautiful things about Kanpur is its affordability. People are very cooperative there.

DB: Did you spend your time in the campus or went outside too?

SR: Even outside the campus, ChinaTown and other places were good. Bada Chauraha and other places were great to visit.

DB: As you mentioned that you were interested in sports, so did you enjoy any sports in your childhood?

SR: Actually, back then, it was rare for me to stay at home. I would always be out with my friends, playing cricket, badminton, tennis, running, and other athletics.

AV: As you mentioned, one of your advisors was AK Lal. Who was the other one?

SR: The other was Prof. Shashank K. Mehta from the CS department, and both of them worked in Graph Theory. I had actually attended a course under Prof. Mehta and asked him if I could do some reading work with him; that was the way I worked with him. I joined under AK Lal and he was very supportive and told me that I could take Prof Mehta as a co-advisor too.

AV: What were the works they were doing back then?

SR: Mainly Arvind Kumar Sir was working on Algebraic Graph Theory, in particular related to Laplacian matrices of trees, which is called the second smallest eigenvalue connectivity. He was working on that along with his supervisor and another student. His supervisor was Prof Bapat. He had many students, two of them were AK Lal and Prof Sukanta Pati from IIT Guwahati. I was interested in that too but the problem to me was assigned by Shashank K Mehta and I continued that problem as I had started reading with him while doing his course. Even AK Lal helped me a lot with his input while writing the paper.

AV: Do you remember the problem?

SR: The problem was to consider a graph X, i.e. with vertices and edges, and then consider an adjacent matrix A, and take A^2, A^3, and so on, and then collect them and observe how the entries change, or if there is a common pattern. For example, if you take a photograph made up of pixels, one can observe many pixels are equal, then the question arises that how many of the pixels have equal values? So if I take A^2, A^3, and so on, then how do I predict the pixel value at different positions? Like that, if I now take a graph and take its adjacent matrix, then I can find its powers and check how the entries change. Now that is the broader view of the problem, but the question was if I could predict the values.

AV: Were there any other people in the Department of Mathematics and Statistics at IITK to whom you talked a lot?

SR: Now the thing was that since you have a course work, after joining you have a year to do the course work and finalize your area of research and advisor. Besides that, there was also an exam one had to qualify to get started with the PhD. And the institute pays you to do some coursework and TA duties, so I had a few, and I worked under many people.

AV: What were the courses you were taking in that one year? Do you remember anybody that you had worked under as a TA?

SR: I took some compulsory courses, such as Algebra, Analysis, and Differential Equations, and some elective courses such as Algebraic Graph Theory, and Approximation Theory. There are some courses for Engineering, like MAT101, where all engineering students would come, which were approximately 700, so a room called lecture hall-7 was used, since it had the capacity to hold more than 700 people. These courses are generally taught by 2 faculties and there are tutorials for 40 students at a time in the Tutorial Complex. So, many times, there were Prof P. Shunmugaraj, Prof Piyush Chandra, Prof Bahuguna, and many other professors, under whom I worked. Sometimes Algebra was taken by AK Lal, and a few times by AK Maloo Sir. In fact, I took a Commutative Algebra course from him and even worked under him as a TA.

Actually, my supervisor and AK Maloo Sir were close friends. So sometimes Lal Sir's students would be Maloo Sir's TA and vice versa. Once what happened was- while I was a TA under Maloo Sir, I had corrected a question by a student that was attempted in a different way and I didn't read properly so I gave very less marks, then the student went to Maloo Sir, and then Prof Maloo asked me to come to his office and explained me what happens if students get less marks than deserved. The student lost confidence hence, he would come and check the papers every time we conducted exams. So one should be very careful while going through answers and solutions, and this incident influenced me a lot.

DB: How was your PhD Defence? Who was sitting on your committee? Any questions you remember from that committee?

SR: Prof R. Thangadurai, from HRI, came for the PhD exam. It went well. After the defence, my supervisors told me that I did a wonderful job. But of course, I also made enough efforts to try to make it clear to everyone and tried my level best to think about how to present things, especially while going for walks in the morning. The committee members asked me some technical questions, like the future aspects of the research and other similar questions. There is an area named Statistical Design Theory, and as I have mentioned the problem I was tackling- so, as I was trying to attack this problem, I visited the IITK library several times, and once I observed a book on Statistical Design Theory that fascinated me and I came across something called 'association schemes' and figured that the problem was related to this. It deals with strongly regular graphs, among other things, and R.C. Bose works in this area. So, I then started working on it, and the questions I was asked were majorly focused on this. I was mainly confined to regular graphs, distance regular graphs, etc., so they asked me to find out what happens if the graphs are not regular and there was another question regarding application.

So in my area, what one does is- take a graph X and then deal with the adjacent algebra of X, i.e., you take a square and so on and you take the polynomial algebra of A. Like $f(x)$ defines a set of all polynomials in x, and coefficients from the fieldX, instead I had it for A- the Adjacency Algebra. So they asked me about the properties of the same and their applications.

PT: You mentioned that there was a professor from HRI for your PhD defence. Have you ever visited HRI?

SR: Yes, I did visit HRI, actually because of Prof Thangadurai himself. I visited for the first time as an observer, which was for a week and I was very happy, it is a wonderful place. And later, he himself asked me to come as a tutor for AFS. AFS is an Annual Foundation School for PhD students. The participants appreciated me and wrote very positively about me and following that, I have visited several times there.

DB: Your paper received first position for oral presentation on Respectable Graphs, at a national seminar held at Brahmanand College, Kanpur on February 12, 2011. Can you share what inspired your research on this topic and the key insights from your presentation?

SR: Actually, until then, I didn't have a paper, and this was when I presented one of my works which was not a part of my thesis. I defined it and presented it there and luckily I won the first position for that. It was a very happy moment for me as my research got some recognition and later I submitted the paper and it got accepted. At present, I may agree that it is not that good an achievement, but I did feel very proud at that moment. I will never forget this and the city of Kanpur.

The work was about two graphs, say X and Y are said to be respectable if one is a polynomial in the other- this means that the Adjacency Algebra of X is equal to the Adjacency Algebra of Y. So, if you take two graphs, X and Y, and let A be the Adjacency matrix of X, and B be

Fig: Satyanarayana Reddy speaking in a workshop on *Diophantine Equations* at Himachal Pradesh Unviersity in 2014. By Satyanarayana Reddy

the Adjacency matrix of Y, then the set of all polynomials in A and the set of all polynomials in B- these two algebras are the same and then we say that these two graphs are respectable. This sort of work was done earlier by Prof Beezer and others and they defined something called Orbital graphs among other things. So I defined this and when it got accepted, I felt very happy. I do believe that it was a good achievement for me.

PT: You mentioned that MTTS has drastically changed your opinion on academia. How did you become a part of MTTS? How would you describe MTTS for the young scientists in the making?

SR: It was certainly my bad luck that I was not aware of MTTS up until my PhD. So I would definitely tell everyone who knows about MTTS to enlighten others about it too. MTTS has a different methodology, the one that is actually required for thinking in Mathematics. So in particular, MTTS encourages you to think more, to first think and then write, it focuses on allocating more time to thinking. Generally one is in a hurry in trying to complete the syllabus but instead, MTTS focuses on taking a particular topic and asking several questions on that itself. The more questions you ask, the more you learn about it as well as its connection to other areas.

One must not be in a hurry. So if you study a definition, then ask several questions about that definition before going further, do not stick to the textbook, and collect more examples, and you will learn that whatever you have asked is what has been asked in the textbook questions. This makes one enjoy it a lot, and is required for every mathematics student. You must not follow somebody, and believe in yourself, and then ask more questions- this is what MTTS

Fig: A celebration with B. Sury and others. By Satyanarayana Reddy

also emphasizes. How I got into MTTS was that, on the feedback forms, the participants had mentioned my name. In 2013, it took place in NIT, Surat and I again went as an observer and even there the participants wrote about me. The interaction with the students helped me.

Once a professor had to go to IIT Patna for MTTS but he couldn't and I was told a day before by Prof Santhanam to go the next day, and since it was a good opportunity, I accepted and went. In the beginning, I had joined just for the one-week programmes. Later, I got involved more, and I myself conducted an MTTS camp as a local coordinator in SNU itself, in 2015 & 2016. I was also a part of the faculty.

PT: How was the celebration of Prof Kumaresan's 74 birth anniversary organized by the Curry Leaf Club? Could you tell us something more about him as well as the club?

SR: So MTTS alumni have a club[2] which was started in 2020 during Covid time. One of the things that happened good in that period was the comfortability with online meetings. The club was started with the intention of giving back. In 2020, some students established contact with Kumaresan Sir and started the Curry Leaf YouTube channel[3] and they started interviewing him and other people and the channel flourished. Later they started the society, and the MTTS trust tried to help them and I was made one of the Faculty Advisors. The club mainly targets MTTS alumni to give talks and share their experiences, and there is also an MTTS summer Programme, and the participants who had given the presentation there could

[2] https://sites.google.com/view/curryleaf/
[3] https://www.youtube.com/@curryleaf6377

also give a virtual seminar presentation on this channel. Now, there are so many talks that are given by students on the Curry Leaf Channel. There have also been many Panel Discussions, and several different types of activities have been conducted. This is definitely an inspiration for many people to start a society and imagine what wonders could be done if people met in-person.

PT: How was the name 'Curry-Leaf' chosen?

SR: Actually Curry Leaf is a technique that Prof Kumaresan uses. So while proving, in Analysis what we generally assume is to add something to both sides and then subtract it in the end. So when we have to compare, let's say A with B, to show that $A<B$, then it would be difficult, hence what we would do is take a C and show that $A<C$ and $C<B$, then this would imply that $A<B$, by the transitive property. Sir referred to this and the Triangle inequality as the Curry Leaf technique. While using curry leaf in dishes, we use it for the flavor and put it aside while eating, so Curry Leaf refers to that.

DB: What are your current work interests?

SR: I am continuing with Algebraic Graph Theory, in this you construct a graph and then take its Adjacency matrices, Laplacian Matrices, Distance Matrices, Eccentricity matrices, etc., but now if the graphs are being constructed from Algebraic structures, then you take a group G and the vertices are the elements of the group and there is an edge between them if they commute or one is the power of the other or one generates the other, etc., in particular, I work in generating graphs and the cyclic subgroup graphs- on which the vertices are the cyclic subgroups and there is an edge between them and one is contained in the other, i.e., there is an immediate containment like Hasse Diagrams in Lattice theory. So most of my work focuses on that for now. Earlier, I also worked on Circulant Graphs, mainly, they have applications to Signal Processing, and this work is still being continued. Also, I work in Adjacency Matrices, which are non-negative matrices with entries 0&1. So we take a non-negative matrix and figure out if they are totally positive or totally non-negative.

AV: As you have mentioned a lot about interactions in your journey. We would like to know what you think about 'thinking' and 'explaining' when it comes to mathematics.

SR: Of course in order to explain something, one needs to think, and hence both are necessary. But it is always better to learn more by discussions and explanations. I suggest that everyone must discuss, give talks, give seminars and hence one must keep adding to the knowledge on the topic they are interested to give a talk on. Collecting information would make a very good expository talk on that particular topic. So before giving the seminar, one needs to think a lot about the presentation, and try different patterns. Keep coming with fresh ideas and in case you are unable to think, revisit the concept, collect more information and then again start thinking and that helps you improve. Even after the talk is complete, you must keep on thinking and improving. It is better to have some sort of 'treasures' with you, some topics that you are ready with.

DB: Is there any mathematics textbook that every undergraduate must read? And why would you refer to them?

SR: It depends upon the requirements of the students. For Undergraduates, at this juncture, the foundation book by Prof Kumaresan Sir, Bhabha Kumar Sharma and Ajith Sir is good. Mathematics has a language, we use something called quantifiers in there, definitions, and the way of stating definitions properly- so the foundation book helps you with all the subjects that you must have done earlier. There are several other books, for leisure reading, like 'What is Mathematics', among others. Now depending upon the student's interest, for someone interested in Algebra or Linear Algebra, then Linear Algebra done Right by Sheldon Axler is a good book. Hence, every area has a different good book.

Another suggestion by me would be Elementary Number Theory by David Burton which is more like a novel and it motivates one very well. It is a book that one can complete on their own and it gives one a lot of confidence. In Elementary Number Theory, the questions are very simple and the proofs are small, which helps gain confidence.

PT: A career in mathematics is not seen with excitement among younger people of India in comparison to other streams, what do you think we are missing here?

SR: First of all, a student must listen to their heart and realise what they are naturally interested in and they must discuss with other people but must not blindly listen to them, and instead listen to their heart. Coming to Mathematics- it's just like a mother. Mathematics is always with you; if you believe in mathematics, then it will take care of you. Using mathematics, one could even earn by providing classes, as I did. For girls who get married and there's some financial constraint or something goes wrong then at least they would have Mathematics to get some earnings.

For other areas, every subject needs mathematics, there is no question about it. Recently, if you look at Coding Theory, or Cryptography or Cyber Security, E-business, Online Marketing, Movie Making, Computer games, Medicine, Bioinformatics, Complex Networks, etc.- all require mathematics. For example, if you go to networks and ask what are all the places that Air India will connect, or Indigo will connect etc., all of them can be expressed by graphs and there are so many different areas that can be connected using Mathematics, i.e., via mathematical modelling. So Mathematics is always better to learn, even if you are not pursuing a career in it. It helps you with other careers as well. I would say the same again, if you believe in mathematics, then Mathematics will be with you, wherever you are or whatever area you choose. Mathematics will help you excel in that field, be it banking, insurance, or product sales, all require a mathematical model. Climate change too requires mathematical modeling. Whatever area one takes, mathematics is definitely required- so, if you believe in mathematics, it will take care of you.

PT: Do you have any hobbies or interests outside of Mathematics, and do you think that those help you in drawing inspiration?

SR: No, I do not actually have anything like that. Teaching, and interacting while doing mathematics is mostly what I do. If I have some works in signal processing, then there the nth roots of unity have a lot of importance, for example, fast Fourier Transform, Fourier Transform, and Discrete Quotient Transform- they are all related to the nth roots of unit., which fascinated me, so I have some good work in that area.

DB: How has SNU been for you? How do you balance the dynamics between research and teaching there?

SR: At SNU, we have a mathematics major course, for BSc students, which is a four year program. It is basically BSc research., and the students are completely confined to mathematics, of course, they have some UWE (University Wide Electives), and CCC courses but the major chunk is done in mathematics. In the first two years itself, they complete Calculus I, Calculus II, Algebra I, Algebra II, Linear Algebra I, Linear Algebra II, Analysis I, Analysis II, and hence they are already equipped with some essential material. Now, in the next two years, they are ready to explore higher Mathematics, hence they come to ask us if we can offer them Commutative Algebra, Galois Theory, Coding theory, Combinatorial Design Theory, Cryptography, Measure Theory, Mathematical Finance, Combinatorics, Algebraic Combinatorics, Algebraic Graph Theory, etc.

I actually have six journal papers up to now with my undergraduate students. I could tell you a few stories about them. When I was taking a Number Theory course, one of the students asked me about the Euler-phi function, that is, the number of numbers that are relatively prime to n. In the class, a student asked me what if I do not want to count all the numbers, but the numbers that are relatively prime to a given number and are composite, i.e., the non-primes that are relatively prime to the number. I did not know the answer then, but when I came to my office, I wrote a program and collected the data, and we sat together, observed the patterns, and wrote a paper.

With another student, I worked on graph theory connected to analysis; one of the other works was LCM of n-tuples and Alternating Sign Matrices. Luckily at SNU, we have a course called Undergraduate Seminar and that course has no syllabus, one can explore to whatever depth they want. They can take a topic and read and this is a very nice break for them. Up until then, they have been reading some syllabus textbooks, and this particular course lets them choose their own topic without any pressure. They can even take up higher mathematics, like Algebraic Geometry among others, and can learn on their own. They can even take a particular topic and research it. In one of these courses, I gave a student the topic of Alternating Sign Matrices and there was a lot of research and learning, and then we finally published the work.

There is another thing called OUR at SNU, which refers to Opportunities for Undergraduate Research. The students, alongside doing the courses, can take up a topic and do research with a professor. This year too I have a student under me and I have given him the topic of Perminants. As in Determinants, we add and subtract alternatively, in Perminants, one adds all of these. I have two papers with the student as of now.

So at SNU, I am doing research with the students as well as teaching here. It is a very nice platform for me, and I was fortunate to join here. I have the flexibility of visiting places and giving talks as well as being a resource person for MTTS and conducting programs there. It has been a great experience here at SNU so far.

AV: In mathematics, we always tend to lose touch with the ground and become abstract enough to sometimes become 'not understandable' and even 'irrelevant'. How does it look for you? Do you believe that rigor can be balanced between intuition and abstractness?

SR: Actually, it is different for different people- some people are very good with it. As Vivekananda says- 'Concentration is the key to knowledge', and he defined something called, 'the degree of concentration'. So, some people can grasp something very quickly, and their degree of concentration is very high. Like I feel that Prof Kumaresan's degree of concentration is high. Person to person, the degree of concentration changes. Some people can write the first step and skip directly to the fourth, while some have the power to directly move to abstract thinking. We need to understand that people are different; some can observe the patterns and come up with something abstract very quickly, but for me, whatever abstractness is there, I go slow and then observe. One can slowly work out, observe, and then conclude with something. It might take time, one can slowly work, and it may be frustrating but please allocate time- even abstract can be made concrete.

In that manner, Prof Kumaresan Sir's approach of teaching helped me. Some students feel that I am understanding, but if you go to abstractness without a proper ground, then it becomes dangerous. Some students feel they understand but they haven't yet, and then they feel overconfident and always talk about jargon. Hence, one must be careful. There are a few who are great and understand things fast, but comparison must not be done.

PT: What potential corrections should be made in an undergraduate program of Mathematics?

SR: The syllabus I think is okay, the more important thing is to focus on the delivery. The way the students think of themselves is more important. The sort of thinking where one can observe things by themselves is more than enough. Regarding the modifications, it is better that we are using technology, and hence at least in a semester, two courses must have Sage components with them- Sage, Python, or any other programming component. So, if one studies Number Theory, then one must also be able to write programs. Since the future is about theoretical computer science, everything they learn, they must be able to convert it into computer programs. Algorithms are nothing but proofs and hence these help us to develop better, optimized algorithms, and that also helps one to realise the need for different proofs to the same problem. One does not blindly follow somebody and just starts, they need to sit and think- that thinking is needed and writing programs help with that.

In conclusion, the courses need to have a programming component, not every course but at least one or two must, however it does become difficult to implement with a huge number of students. The other main focus is to change the examination system- mainly, the way of asking questions. Right now, major focus is given on writing examinations, which has to be changed- the evaluation pattern must be changed and there should be a continuous evaluation, students need to know where they are lacking continuously, and that cannot be done with one exam.

AV: People are afraid of asking trivial questions. How do you encourage them to ask trivial questions, i.e., in a math lecture, or seminar, or even in a regular class?

SR: One should not be afraid, or care about what other people think, and hence one must compare their progress only with themselves. If you feel any difficulty, you could talk to your friends or directly to the faculty and hence slowly start asking questions in the class too. With one person, even others would gain the confidence to ask. Some would think that these questions are very easy, but sometimes it isn't easy- not just for you, but for other people too, and hence when you ask questions, you also help others. And a good way to tackle that would be to form a group and then ask questions, which helps a lot with discussions. You must take a definition, try to collect as many examples as possible, look at the patterns, and automatically, the questions will come to you.

DB: Mathematics, for some people, acts simply as a tool, and for some people, it is a form of pure thought. Which culture is weaker in the Indian education system according to you?

SR: Tool is actually the wrong way to mention it. Mathematics is helpful in all careers as I said. Earlier, even Physics majors, Engineering students, etc.- all of them were actually very good mathematicians. Engineering is basically an application of science- so they need to understand the science better, to apply it. Just learning and doing it for a particular problem doesn't clarify the topic in your mind, but if you actually read and ask questions and understand the subject thoroughly, only then do you understand that. Do not underestimate whatever the concepts you are learning, and give time, ask questions, and sooner or later, it comes to you. Hence, Mathematics must not be treated as a tool.

PT: We, at Anveshanā, aim to understand and celebrate the beautiful connection between scholarship and human thoughts. What are your views about the importance of human thoughts in the subject of Mathematics? How important it is- for a student, to develop their own independent inquiries and come up with novel and original

ideas? Lastly, do you have any central advice for young people who want to have a career in mathematics?

SR: First of all, the name that you have chosen, Anveshanā, is a great name. One of my favourite movies is Anveshana- it is a Telugu movie. Coming to the question, look at the subject, Physics, Biology or Chemistry, usually, we are talking about the things that are already there. If you take History, then too, we are talking about things that have already happened, of course, it helps us in many ways. In Physics, we try to understand the already existing things, and in Zoology- you study about animals, etc., but Mathematics is a subject which is created by man and it is already about thoughts, and it was created to solve all these problems. I would repeat again, Mathematics is just like a mother. Even during this interview, the signal transfer takes place; while paying at a shop with the QR code, the coding theory is applicable there; in the transmission of encrypted messages, Coding Theory is applicable, and in Cyber Security too- all these applications come from Number Theory, Group Theory, etc., in Mathematics. As I have already pointed out, Mathematics helps one to think, which helps in every area. Therefore, one must allocate time to mathematics, and read and discuss with their peers. Undergraduate students could discuss with highschool students and try to explain things to them.

Article

Quantum Complexity

by Devang Bajpai

What is Complexity?

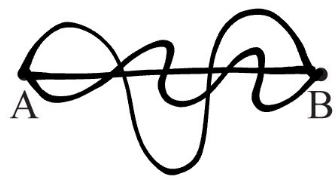

Fig: Shortest Distance between two points A & B Devang Bajpai

Let us start by asking a simple question- what is the absolute smallest path between A to B? Complexity is the absolute minimum number of steps needed to go from A to B. Does it significantly matter how many steps are involved? Generally speaking, it doesn't. Considering another example, let us discuss the complexity of classical states of N bits. Now, N bits refer to the binary units that are in a computer, i.e., $\uparrow$ and $\downarrow$.

$$\text{A simple state: } \uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow \ldots$$

I will take the simplest state to be the state in which all the bits are up. And here these are simple but universal gates. Now, a gate is simply an operation that you can do on these bits, and it involves only small number of bits. Gates can be assembled into quantum circuits in the manner shown in Fig. 6.1b. The simplest gate would be the 'flip gate' in which you simply flip one bit, and that's again, a simple but universal gate. The maximum number of gates that you will ever need to get the final output is no more than N. So the maximum classical state complexity will be N.

Feynman[1] pointed out that the quantum state can be more complex than the classical state. A quantum state can be thought of as linear superposition with coefficient ψ_i

$$|\psi\rangle = \sum_{\text{classical state}} \psi_i |classical\rangle$$

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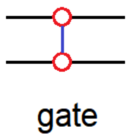
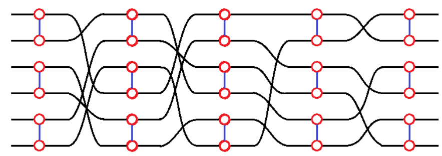
[1] R. P. Feynman, Simulating Physics with Computers. International Journal of Theoretical Physics, 21(6), 467–488, 1982.



(a) A gate acts on an incoming state of two-qubits to give an outgoing state. Leonard Susskind, arXiv:1810.11563v1



(b) Standard circuit architecture. Leonard Susskind, arXiv:1810.11563v1

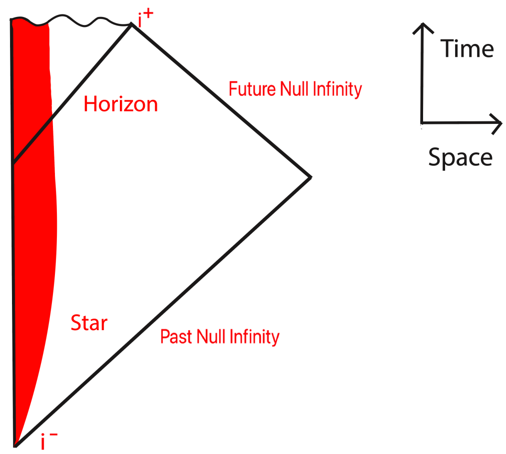
$$\psi_i = 2^N \text{complex Amplitude}$$

For N quantum bits there are 2^N classical states[2] and that is the reason that quantum states are computationally hard. Moreover, other than complexity of states, there are also the complexities of unitary operators and so forth, but I will not get into those.

The maximum quantum complexity that can ever happen is instead of being N, as it is for classical physics, it is e^{2N}, and almost all states have close to maximal complexity. Suppose you are given a quantum state, and you want to compute quantum complexity. To answer this question you can go back to the question of what is the minimal distance on geometry from one point to another. It is easy to find geodesics but it's very hard to know whether the geodesic you found is the minimal.

So while complexity is a well defined thing it can be relatively small, since it doesn't have to be very big. It is generally very hard to compute. The lesson from all this is that quantum complexity is a very peculiar quantity, and you can't tell the difference between a large value and a small value; it's ethereal. And for computer scientists it is almost imperceptible, but this makes it even more interesting- like energy, entropy,and density, simply by the virtue of property that they are imperceptible.

In 2014, *Leonard Susskind*[3] conjectured that the volume of the interior of a black hole is the complexity of it's quantum state. So for physicists, it's the interior of a black hole.



Penrose Diagram for a Black Hole Edward Witten, arXiv:2412.16795v1

Black Holes: Volume ∼ Complexity

Talking about black holes, according to Hawking and Bekenstein, the entropy term is given by the area of the black hole with a numerical factor,

$$S = \frac{A}{4G\hbar}.$$

The second law of thermodynamics says that entropy increases with time but until you get to the thermal equilibrium (and that takes a very short time after that black hole is completely static, at least from outside). If we plot the area and entropy we get a similar plot like the complexity graph Fig.6.2.

In a black hole, the volume of the interior increases for a very long time and it goes much like entropy. Now complexity grows for a long period of time until it hits the maximum possible complexity of a quantum system. For a solar mass black hole, the time that it takes to equilibrate is 10^{-3} seconds. Technically, the process is called the 'scrambling'.

In the Penrose diagram of a black hole, the complexity of the system continues to evolve and grow for an incredibly long time. This growth resembles the behavior of entropy but occurs on a vastly larger timescale—on the order of $\exp 10^{78}$ years for a solar mass black hole. During this immense duration, the black hole interior's complexity increases steadily until

[2]A Complex Amplitude describes the magnitude and phase of a wave or quantum state. It is represented as a complex number, where the real part corresponds to the wave's magnitude and the imaginary part encodes its phase.

[3]L. Susskind,"Computational Complexity and Black Hole Horizons," Fortsch. Phys. 64 (2016), 24-43 doi:10.1002/prop.201500092 [arXiv:1403.5695 [hep-th]

quantum fluctuations eventually accumulate and halt this growth. This phenomenon is depicted in figure 6.2.

Complexity Growth

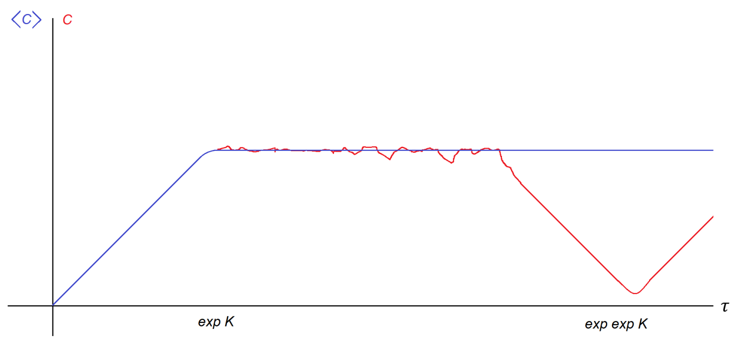


Figure 6.2: Evolution of complexity with time. The ragged red curve is the evolution of a specific instance of an ensemble. The smooth curve is the ensemble average. Leonard Susskind, arXiv:1810.11563v1

Now to determine how the complexity grows for some amount of time, the number of gates will almost continuously be the minimum number of gates that it takes to get where you're going, so it follows that complexity begins to grow until it hits the maximum possible complexity of a quantum system, which is exponential like entropy.

This looks very much like the second law of thermodynamics that is, the growth of complexity is governed by a principle analogous to the second law of thermodynamics, which we call the *second law of quantum complexity*.[4]

Concluding Remarks

In conclusion, quantum complexity is not merely a theoretical curiosity, it is a very important field of information theory and theoretical physics.[5] Its implications extend from advancing quantum computing to unraveling the mysteries of black holes and spacetime.

[4] A. R. Brown and L. Susskind, "Second law of quantum complexity," Phys. Rev. D 97 (2018) no.8, 086015 doi:10.1103/PhysRevD.97.086015 [arXiv:1701.01107 [hep-th]].

[5] See L. Susskind, "Three Lectures on Complexity and Black Holes," Springer, 2020, ISBN 978-3-030-45108-0, 978-3-030-45109-7 doi:10.1007/978-3-030-45109-7 [arXiv:1810.11563 [hep-th]]

Article

Women of Sciences

by Purnima Tiwari

For a person in Science, especially mathematics, one must have come across a well-known and appreciated book, titled 'Men of Mathematics' by E.T. Bell, which is a beautifully penned piece of mathematical literature and talks about the lives of men from the history of Mathematics. But what impresses one is that the book doesn't limit itself to their lives in academia, we see their lives overall, looking beyond their works, into their interests, their hobbies, where some such as Archimedes fought wars for their kings, or some like Galois revolutionized the course of mathematics at a tender age of 19, and some like Descartes, who laid the groundwork for Geometry but however became a slave to his poor health.

When talking about Women in Sciences, the word *in* limits itself to the multitudes of dimensions which *of* can offer. Through this article, I aim to bring to light the women who have emerged elegantly in the diverse fields of Sciences, against the prejudice of the time.

Ada Lovelace, (1815-52)

Lovelace was born to Annabella Byron and Lord Byron in the first year of their marriage. Annabella was herself mathematically talented and was called *'The Princess of Parallelograms'* by Byron. Byron and Annabella's marriage was short lived and Lovelace didn't get her fair share of experiencing the love of her father.

Her interest in mathematics was initially pursued by Annabella, who was concerned about the insanity of Byron's nature that might have been of interest to Lovelace because of his intellect, but towards the end, she had developed a keen interest in the discipline for herself. Annabella was determined to make Lovelace self-dependent and at a very young age, exposed her to the beautiful world of mathematics. Lovelace's mother ensured that she was given the best education possible of the time, and the famous logician De Morgan was also one of the tutors of Lovelace when she was 8.

Lovelace, by the age of 12 had developed a zest for Mechanics. She had somehow become fluent in writing papers, and never missed an opportunity to do so, especially when she encountered Charles Babbage at a reception at the age of 17.

Babbage was in his early 40s, and didn't wait much longer before revealing to Lovelace, the prototype of the working model of 'The Difference Engine'. Ada was highly impressed and

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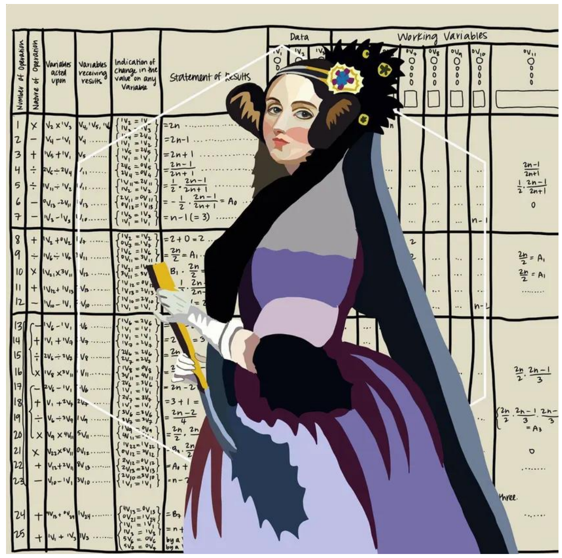

Fig: Ada Lovelace Courtesy of Sumana Mahata

rather intrigued by his ideas and this led to the start of a correspondence that the mathematical and the computational world has undoubtedly benefited from.

Lovelace was quite interested in Babbage's second project, called the 'The Analytical Engine' in which she had contributed a great deal. This project was supposed to be a major step-up from the first one, since it could do more than just simple calculations and solving polynomial equations of lower degrees.

Lovelace had designed a program to generate the Bernoulli's numbers using the Analytical Engine and had also imagined a vision of a working machine that could process graphics, such as musical notes, letters, images, etc. And perhaps rightfully, the programming language 'Ada' is named after her.

What fascinates one about Lovelace is that a century before the first programmable computer was designed, Lovelace had written a computer program. She had built connections with people such as Charles Babbage, Charles Dickens and Michael Faraday, and had achieved much by the young age of 36, when she died of cancer and was buried next to her father's grave.

Lovelace would describe her mindset as *'poetically scientific'* and expressed her views on imagination boldly, as-

"What is imagination?
It is a God-like, a noble faculty. It renders earth tolerable, it teaches us to live, in the tone of the eternal."

The 'Father of computer himself' named her as *'The Enchantress of Numbers'* and her life has been documented in a novel by Jennifer Chiaverini, by the same name- 'The Enchantress of Numbers: A Novel of Ada Lovelace'.

Marie-Sophie Germain, (1776-31)

Germain was a French Mathematician, Physicist and a Philosopher, who was determined to work independently throughout her life, due to the prejudice against her gender. She had no former education and taught herself all the necessary mathematics to catch up on the notable works of the time. Her interest in studying was demotivated and protested against by her parents, only to later realize that nothing could stop Germain. Her parents went so far as to not provide her with any heat at night so that she wouldn't dare to leave the room, but even that did not stop Germain and she continued to *sneak* into her father's library and read.

Germain, at a very young age was exposed to political and philosophical discussions and she was around the age of 13 when she used to visit her father's library and pass the time reading. Coming across Étienne Montucla's *Histoire des Mathématiques*, Germain read of his account of Archimedes who was so indulged in doing, pursuing, or rather *living* mathematics, that he would forget to eat and drink and at the end of the day, was killed at the hands of a Roman Soldier because of the same. She was moved by this and henceforth determined to be *'living'* mathematics, for the rest of her life.

Germain never married, and neither was she honored with a position in academia but despite that, her father eventually realizing her passion for the discipline, continued to support her financially throughout her life.

All that she knew about the discipline, until the age of 18, was self-taught. She was tackling concepts like differential and integral calculus, during the 'Reign of Terror', which we believe, brought her comfort during those times. Her house was a few feet away from the land of bombardments, during the time of the revolution and indeed, nothing could stop her.

Germain established connections with a professor at the *Ecole Polytechnique*, a new university at the time, and despite the restriction for women to study, she managed to obtain lecture notes from the professors, and even submitted an assignment that caught Legendre's eye, primarily because of it's originality. However, she had aliased herself as *M LeBlanc*, a student that had formerly studied there but passed away. Legendre was marvelled at the brilliant mind of LeBlanc, and emphasized on meeting *him*. However, he was more amazed when he figured that M LeBlanc, was actually a *'she'*, and then introduced Germain to his circle of scientists and mathematicians. Germain's correspondence with the academic world doesn't stop there. Later, she got interested in Number Theory, specifically, Fermat's Last Theorem, and corresponded with Gauss under the alias and even he was amazed at the marvellous mind of the young M LeBlanc, and more so when he figured that it was Germain, a young lady

Figure 7.1: Fig: Portrait of Sophie Germain, circa 1790. Wikipedia

behind the letters. Gauss wrote about her to Wilhelm Olbers saying,

"But when a woman, because of her sex, our customs and prejudices, encounters infinitely many more obstacles than men, in familiarizing herself with their knotty problems, yet overcomes these fetters and penetrates that which is most hidden, she doubtless has the most noble courage, extraordinary talent, and superior genius"

Germain made strides and had a detailed plan of proving Fermat's Last Theorem, during which she stated and proved the Germain Theorem, however her work remained largely unnoticed as 'Lengendre's footnotes' in his publication.

Germain worked throughout her life on several notable topics. A handful of them that still remain at the top of the list are Number Theory, Elasticity Theory, Germain's Prime, Germain's Theorem and specifically, her work on Fermat's Last theorem has brought to light, centuries worth of opportunities for mathematicians to work for in number theory. Rightfully named, she is also known as the *'Princess of Mathematics'*.

Besides mathematics, she had made great efforts in the field of philosophy, aiming to classify facts and generalize them into such laws that could eventually form a system of psychology and sociology, which at that time, were the emerging fields. Despite receiving the unfortunate news of having breast cancer in 1829, Germain continued to pursue her works among which one of them led to the discovery of the laws of equilibrium and movement of elastic solids, for which she remains largely unappreciated to this day.

Germain remains a name untouched in the history of mathematics, so much so that there is a prize awarded in her name by the Foundation Sophie Germain, conferred by the Academy of Sciences in Paris. This award honors French mathematicians for research in the foundations of mathematics.

Before her death due to breast cancer, Germain was setup to receive an honorary degree from the University of Gottingen, but she never could, because of her untimely and unfortunate demise. She has even published two philosophical papers, besides the disciplines of mathematics and physics.

Her thoughts on algebra and geometry are reflected in this quote by her-

"L'algèbre n'est qu'une géométrie écrite; la géométrie n'est qu'une algèbre figurée."

which translates to-

"Algebra is nothing but geometry, in words; geometry is nothing but algebra, in pictures."

Marie Curie, (1867-34)

Born as Maria Salomea Sklodowska (Manya, as she was affectionately called), in Warsaw, the Kingdom of Poland under the rule of the Russian Empire- she was a Polish born French chemist and physicist who conducted pioneering research on the theory of Radioactivity. She was born during the times when it was hard for the Polish to survive, due to the oppression by the Russians, and even speaking in their native language would have gotten them into trouble of a magnitude which is hard to imagine.

The first woman to win a Nobel Prize, the only person to win a Nobel Prize twice, and the first in the string of the Curie family legacy of five Nobel Prizes, Manya had a fair share of hardships and loss that she dealt with, starting from her childhood, when she lost her mother and elder sister by the age of 10. Her mother had resigned from her position as a teacher at a school that she taught in, after Manya was born. After the loss of two very close members of her family, Manya decided to follow the footsteps of her father who was an atheist and lost her trust in Catholicism and rather developed an agnostic approach towards religion.

Manya graduated with honors from her high school, however she fell into a dark pit of depression, since she had reached the pinnacle of women education during those times in her nation. Her brother went on to pursue medical sciences, however her sister- Bronya and she could not pursue studies any further in their own country, and hence they had become a

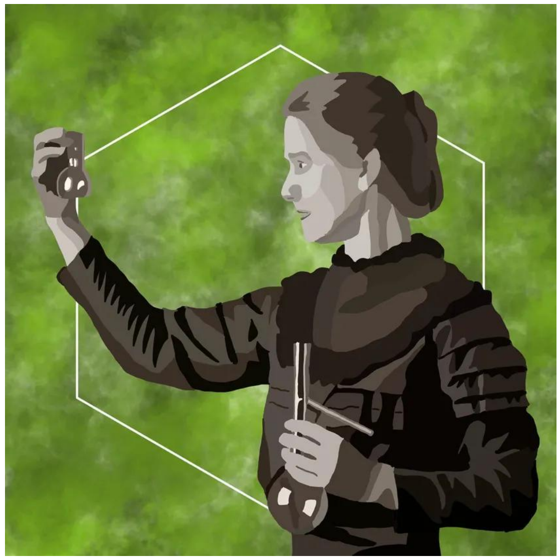

Fig: Marie Curie Courtesy of Sumana Mahata

part of the *Floating University*, also known as the *Flying University* which used to conduct its lectures at different locations, due to it being a group of Polish-born female students, invested deeply into broadening their horizons and voluntarily propagating their learnings. Manya had realized that such a place would not match the teaching standards of the outside academic world, but she was grateful for an experience like this.

Maria and Bronya then moved to France, where Manya, now Marie, tutored to earn an income to support her sister through her graduate school. However, she later realized that the earning from tutoring were not sufficient to support two people, and then she decided to become a governess. While working to try to substantiate the expenses of teaching her sister and surviving in the new city, she fell in love with her neighbor Kazimierz Żorawski, who was destined to be a mathematician. However, the engagement was opposed by his family due to the impoverished earnings of Marie. They broke off the engagement but stayed in contact for a few years, until she left her job as a governess.

After a while, when Marie's father had started earning a stable income, he supported Marie and her sister through their studies, and hence she had now admitted herself in a university to further her education.

Her early days in Paris had shown her a rightful share of her plight, where she studied as hard as she could, sometimes even forgetting to eat and wearing all the clothes she possessed to protect herself from the extreme cold. She would study during the day and tutor in the evening, barely earning enough to stay afloat, even after her father's help. Marie then went on to pursuing a PhD while working in a laboratory, alongside Pierre Curie.

Marie had started working with Pierre Curie in his laboratory through a mutual connection and eventually they established a bond of trust, resulting into fruitful collaborations in academia and a matrimony. Pierre was much elder to Marie and had lost all hope in love, but after a long time of working together, they had mutually established that a bond like theirs had to be kept safe and together.

They had two daughters, and no matter how hard it got for them to manage time, alongside getting help from Curie's parents, he never asked Marie even once to give up on her career, and he has in fact described that the women he met before her, lacked the passion for their work as she had, almost in synchronization with his'.

In fact, Marie had persuaded him to actually write a research paper as his PhD thesis, since Pierre did not possess an actual degree, however he had several research papers under his belt.

Around the time Marie had attained her PhD, the X-rays and the much weaker Uranium rays were discovered. The X-rays caught the eye of the masses, however she was rather interested in the much weaker Uranium rays, for which she went to receive accolades for, in research.

Pierre passed away in an unfortunate incident and after that, Marie isolated herself from the world, to focus just on her works and her children, and she passed away 28 years later after Pierre did.

Maryam Mirzakhani, (1977-17)

Mirzakhani was born in 1977, in Tehran, Iran, during the Iran-Iraq war which ended by the time Mirzakhani turned 8. The endless support from her family in a country with conservative ideas was definitely the catalyst in her life.

Mirzakhani had a knack for solving mathematical problems, she didn't feel interested herself back then, but found it easy to solve the problems assigned to her elder siblings in their mathematics class. After the war ended, she transferred to an all girls school, but the one that was determined to provide the young women with all the necessary catalyzation which will eventually lead them to become the leading youth of tomorrow that will definitely re-define the course of the world, in tiny though profound ways.

Mirzakhani's first year at the new institute was not a comfortable one, however she realized that if she would work hard, she could make a lot of progress, and so she did. She, in the consecutive years, along with her close friend Roya Behesti, qualified for the international team of Iran, to be appearing in the Mathematical Olympiad, and not much to one's surprise, Mirzakhani bagged the gold medal, while Roya, the silver. In the next year too, Mirzakhani became a part of the team and with a perfect score, won another gold medal. She then went on to pursuing her graduation from Sharif University in Tehran, and wrote mathematical papers, while still being an undergrad, needless to say, she went on to bigger and beautiful

Fig: Maryam Mirzakhani at Stanford University. Maryeraud9, Wikipedia

things there on.

Mirzakhani was a PhD Scholar at Harvard, under Curtis McMullen. She thereafter went on to become the first woman ever to win the Fields Medal in 2014, the most prestigious award in mathematics for her work in 'the dynamics and geometry of Riemann surfaces and their moduli spaces'. She has been affiliated in her past with the Clay Institute, and Princeton too. She served as a mathematics professor at Stanford during her last days, where she continued to revolutionize her areas of research until she passed away.

Born and raised in Tehran, her journey has definitely not been a walk in the park, but her passion for Mathematics alongside literature began at a very young age. Mirzakhani as a child thought that she would rather become a writer, due to her keen interest in literature. She became the first Iranian to receive two gold medals and achieve a perfect score in the International Mathematics Olympiad. She described herself as a *'slow'* mathematician, emphasizing that *'you have to spend some energy and effort to see the beauty of math'*.

Her daughter described her mother's works as 'painting', which was Mirzakhani drawing *doodles* of hyperbolic surfaces on a blank sheet of paper.

Mirzakhnai was diagnosed with breast cancer, a year before she received the Fields Medal, and she passed away three years after in 2017. She however, lives timelessly in the minds and hearts of the people whom she inspired, simply by her presence and aura.

Article

Yoga of Mathematics

by Aayush Verma

Mathematics in its regular sense, at least at school levels, seems like a fair way of calculating practical probabilities and measuring the required events. Though not entirely true, we are often told that mathematics, and its fashion sets limits for empirical and practical personalities. However, mathematics is one of the oldest sciences, and pinpointing the moment when it transformed into an abstract inquiry is vague to answer. Yet, mathematics, even in its purest form is neither trivial nor dismissive in the search for truth. In fact, in my opinion, it is the grandeur of all. It is that spirit that keeps a mathematician alive and produce the hidden and overlooked truth of life and mathematics alike. It seems an inquiry of abstract symbols may not lead us to the truth that an individual begs for, but it does and this is what I argue by the term Yoga in this article. The word, Yoga, is a very different word to be used in an essay on mathematics and I am aware of only Grothendieck using it. Such words have diverse meanings and unclear connotations and I would prefer to not define it too for that task is too complicated for me to do here. But I am prompted to say that words like yoga, beauty or truth in mathematics already carry a great aplomb with which we describe the words, however, this confidence should not *yet* be the final one.

As an inquirer, one can ask, what is that ultimate question that we must answer before we are done with our lifetime or at least, say before our retirement? But, if there is one, it need not be answered and there is no guarantee if we get it as some special or unique resolution which can conclude one's (to use the word) spiritual journey. Then what do we seek as mathematicians when there could not be a general and singular consensus about the truth. That demands a good and mature thought since it does not have 'one' answer. We seek of different objects and we run against different maps. It is the ignorance which cuts the clay of all unity. Before I go elusive in my words, let me get you through a remarkable piece of mathematics that inspires me. It is the concept of a 'scheme' and we will briefly look at it. A scheme is a locally ringed space with a topology and its elements are the prime ideals of the ring R.

A scheme is a beautiful example of the correspondence between geometry and algebra where on one hand, we have elements in the affine space which corresponds to the spectrum of the ring which is the collection of the prime ideals. Such correspondences and dualities are very important for mathematicians and physicists. A scheme is a modern theory in algebraic

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geometry which has its history in the pens of Weil, Zariski, and Grothendieck.

Now let us understand the importance of *dualities* in mathematics and physics and why a professor of such subjects should, once in their lifetime, cultivate an appreciation of this culture within themselves. A duality means a mirror of which two sides are equivalent against a simple language translation. Physicists need not to look at string theory for such dualities, because there are many early examples like wave-particle duality of light and many examples afterwards. String theory, however, has a wide web of dualities and theorists (and mathematicians too) appreciate it. Holography is also a manifestation of duality which enables one to understand obscure language of gravity theory through conformal field theory, or more formally known as bulk-boundary duality. Theorists working in string theory and quantum gravity are often prompted to understand the dualities concerning the theory in hand and such works really help in developing the understanding. We can name a lot many examples but simple ones would be holography[1] itself, or S-duality/Electric-Magnetic duality and mirror symmetry.

For mathematicians too, a duality is a very important principle and should be a guiding light for many if not all. This principle can be seen in the works of great mathematicians like Grothendieck, Serre, Atiyah, Connes, and Langlands. Because note that dualities help us to see sectors of the theory that is not immediately available to us with its sole premise. I must also add that dualities are often unnoticed of their brilliance and innocence during first few looks hence one should be patient in when bridging the two sectors. Few recent examples of dualities can be found in the Langlands program[2] (like the classic Taniyama–Shimura–Weil conjecture).

Another important Yoga in mathematics is to really ask questions that should be **asked** but also the questions that no one is asking. At some point, mathematics becomes personal to a human and an inherent beauty appears in the investigation that transcends the boundary of the discipline, then no matter how trivial or nonsensical the query would be, a mathematician should trust their conscience and ask it confidently without any shame. In a letter of May, 1982 to his friend R. Brown, Grothendieck writes[3]

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[1]Holography has, by now, many definitions and frameworks but the most common agenda is to find a theory which is 'dual' to some other theory, say sitting on the boundary. A very successful holography is AdS/CFT (a duality between Anti de Sitter/Conformal Field Theory) introduced by Maldacena (arxiv:hep-th/9711200), which also solves the black hole unitarity problem in AdS since the boundary theory (in this case, a CFT) is unitary.

[2]One of my favorite interplay between mathematics and physics, alongside Gauge theory, is the recent developments of Langlands program in theoretical physics, like that of the Geometric Langlands Program (arxiv:hep-th/0604151) and the Relative Langlands Duality (arxiv:2409.04677).

[3]Available at https://webusers.imj-prg.fr/ leila.schneps/grothendieckcircle/Letters/ LettersGrothendieckRBrown.pdf.

"The introduction of the cipher 0 or the group concept was general nonsense, too, and for a thousand years or two mathematics was more or less stagnating, because nobody was around to make such childish steps..."

Indeed, mathematics is a child's game and has to be played in a childish manner. We should and we must grow with our mathematical maturity but that should not sterilize or saturate our adventurous instincts. It should only make mathematics a more systematic discipline while still holding the roots of triviality and naivety. But mathematics also requires 'rigor' and that is very important for a mathematician to 'acquire'. For the rigor in mathematics can not be transmitted and a modest rigor can only be encouraged or born into a mathematician. Is mathematics then subservient to the rigor? No, that is far from true.

I speak with a young mind, and the problem with a young mind is its susceptibility to big dreams. As a mathematician, these dreams serve a big purpose of drive and inspiration. A drive, closer to the duality idea, is about locality in mathematics. That enabled Grothendieck to develop the topos theory. The drive of a mathematician is toward knowledge - pure and perhaps absurd to others - but for which no apology is needed from either side.

The thing about the personal and innate creativity of mathematics is that it must be ultimately expressed through intellectual labor and rigor. Beyond the rigor, there always lies a simple and basic thought. The need is of an intense and enduring engagement with mathematics, or any other discipline, that can serve as a chance to challenge ourselves and renew our sense of identity - something which is truly personal to us.

Article

The Art of Generalization

by Arpit Dwivedi

The aim of this note is to look at some famous generalizations done in the history of mathematics and to have an opportunity to see various ideas and motivations behind the generalizations. This note is prepared with a general reader in mind therefore, I don't intend to go deep with the ideas, rather I'll give various examples to illustrate the ideas behind those generalizations.

Introduction

In mathematics, the term generalization refers to extending a concept, idea, principle, or mathematical object (such as numbers or functions) by changing some of the initial assumptions to a broader area of applicability. For example, if we look at the expression $y = mx$, at first, it may seem the family of straight lines passing through the origin. But in three dimensions, it represents a plane; moreover, if we relax the untold, underlying assumption that- the expression has only two variables and generalize it to n variables, we see that it represents a hyper-surface of dimension $(n-1)$ sitting in an n-dimensional space.

This generalization allows us to unite the idea of a family of hyper-surfaces into a single representation, which makes the study of higher dimensions one step closer to comprehend. This is exactly the purpose of generalization, that is to get our initial idea to work on a broader domain so that we can have better tools to understand the world around us.

The aim of this article by no means is to give all the generalizations that have ever occurred, nor it is to teach how to generalize rather, it is to motivate the readers to dive deep into the sea of mathematics by introducing some famous, useful, and elegant examples. To start things off, let me motivate you with arguably the most important generalization ever done in Geometry. That is relaxing one of the Euclid's postulates[1] (which gave us the Euclidean Geometry) to open the gateways to very counterintuitive non-Euclidean Geometry, such as spherical Geometry and hyperbolic Geometry. With these geometries, today we can talk about the shape of the Universe.

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[1] Heath, Thomas L. *The Thirteen Books of Euclid's Elements*, Dover Publications, Inc, 1956.

Examples

Let us now discuss a few fundamental and insightful examples which have occurred throughout the history of mathematics.

The Number System

We started our journey of numbers with natural numbers used as counting numbers and soon realized that something was missing. Suppose a person A has two friends B and C whose homes are in the same street as that of A with A's home in the middle. Both B and C's homes are 25 steps away from A's home. If A has gone 25 steps from his home, we don't know whether he went to B's home or C's home. Hence, what's missing with the information '25 steps' is the sense of direction. To remove this obstacle, the integers were discovered.

Similar necessities were observed and helped to improve the number system further. For example, dividing a cake equally for a number of persons and then asking how much of the cake a person got- created a new type of number, called the rational numbers.

Squaring a number is easy but with the information of rational numbers, the question 'What happens when we take the square root of a number which is not a perfect square?' leads to the real numbers and similarly, taking the square root of a negative number was also absurd, until we came up with the idea of complex numbers.

The Gamma Function

We all know what a factorial is. $n! = 1 \cdot 2 \cdot 3 \cdots n$, where n is a natural number. But what if n is not a natural number? This time, it was not at all obvious how to answer this, and the question had to wait till 1729 for Euler, when he noticed that $n!$ has an integral representation as the following:

$$n! = \int_0^\infty e^{-t} t^n dt.$$

Euler again noticed that this integral makes sense not only for natural numbers, but also for whole positive reals, and it also lets you define the zero factorial. With some modifications, Euler was able to define the Gamma function, which is seen as the generalization of the factorial function, for the whole complex plane, except for negative integers and zero, where it has simple poles. The final definition is as follows:

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt, \quad z \in \mathbb{C}.$$

More about the Gamma function can be found in the book[2].

[2]Artin, Emil. *The gamma function*, Courier Dover Publications, 2015.

From Newton to Einstein

Let me ask you some questions. What is Time? What is Gravity?

⋮

Felt a shortage of words, right? In search of answers to these questions (especially, the gravity one) Newton and Einstein both gave their theory: Newton's laws of motion and Einstein's Special and General Theory of Relativity[3].

Newton's laws fail at instances where very high speeds (a significant portion of the speed of light) are involved, and if the object under study is under the influence of a very massive celestial body (like a black hole or a massive star). While Einstein's equations work perfectly fine in these situations.

Einstein's Theory of Special Relativity is the consequence of an attempt to answer a very simple question: 'Am I in motion?', under the condition that no acceleration is involved. When acceleration is involved, the Special Theory is replaced by the General Theory of Relativity. The fascinating thing about the General Theory is that- if you put acceleration as zero, you will get the Special Theory of Relativity, and if you apply low speeds in the Special Theory of Relativity, you will get Newton's laws.

The Fractional Derivatives

One of the hot topics today for researchers is the fractional derivative. Newton and Leibniz both are credited for the invention of the classical derivatives, but it was Leibniz who used the symbol $\frac{d^n}{dx^n}$ to denote the n-th derivative with respect to the independent variable x. This symbol was immediately accepted, as it was better than Newton's symbols (which were dashes) in the sense that it also tells you what the independent variable is. L'Hôpital, after gazing at his symbol, wrote a letter to Leibniz asking what happens if $n = 1/2$. This was the moment when fractional derivatives were born[4].

Evidently, Leibniz did not have a satisfactory answer for that and said, *someday very useful consequences will be drawn from this apparent paradox.* The question was ahead of its time because the modern definition of fractional derivative includes the Gamma function, which was discovered in 1729 by Euler. Today, we are able to define fractional integrals and derivatives precisely and use them in very wide areas of mathematics. With computers in our grasp, these derivatives are proving to be a better mathematical tool for modeling physical phenomena.

[3] Miller, Arthur I. *Albert Einstein's Special Theory of Relativity: emergence (1905) and early interpretation (1905-1911)*, Reading, MA: Addison-Wesley, 1981.

[4] Miller, K. and Ross, B., *An Introduction to the Fractional Calculus and Differential Equations*, Wiley, New York, 1993.

There are many ways to reach fractional derivatives from the classical one. Let me introduce here the easiest one (in my opinion). Observe that in the following formula for the multiple integrals (called the Cauchy-Euler formula for multiple integrals):

$$I^n f(x) = \frac{1}{\Gamma(n)} \int_{x_0}^{x} (x-t)^{n-1} f(t) dt,$$

the integral on the right-hand side makes sense even if n is not an integer. It is allowed to be a real number and gives out an answer. This gives us an opportunity to define the fractional integrals, and then with the help of composition with integer order derivatives, we can define the fractional derivatives[5].

Conclusion

Through these examples, we noticed that there is no straightforward recipe for generalization. The generalization can be done in different manners, in different situations like, (apart from the above examples,) we might do analytic continuation (which is a way to extend the domain of a function, such that it preserves some nice properties of the function) by power series method.

Obviously, there are many generalizations of the same thing therefore, we look for those generalizations that preserve some nice properties, and most importantly, it must return the same output as the original function, or object, when it is restricted to the condition of the function, or object, before generalization. For example, for positive integer values of n, the fractional derivative operator becomes the ordinary derivative operator.

[5]Podlubny, I., *Fractional Differential Equations, academic Press*, San Diego, 1999.

Article

Hues and You

by Sameeksha Gupta

> *"Everything that you can see in the world around you presents itself to your eyes only as an arrangement of patches of different colors."* - John Ruskin

If we discuss the scientific definition of what colors are. It defines colors as the result of light interacting with objects. When light strikes an object, certain wavelengths are absorbed, and others are reflected. And the ones reflected, determine the color we perceive. But we know that, don't we? One of the most basic understandings of humans is colors. Colors contain various feelings for different people, and they tend to like or dislike different colors on the basis of their experiences. Colors also affect our emotions which is why sometimes we are-green with envy, see red, feel blue, or are tickled pink.

Colors have a profound psychological impact on human emotions and behavior. While the effect of color can be somewhat subjective—depending on personal experiences and cultural backgrounds. Colors always play a huge role in many different fields such as storytelling, music, design, branding, and even retail among others. In Storytelling, filmmakers use color palettes to reflect a film's mood, themes, and characters' emotional journeys.

For example, red might signal passion or danger in a thriller, while blue could convey sadness or an atmosphere in a futuristic film. For example, in La La Land (a movie), colors enhance the emotional depth of the story. Mia's (a character in the movie) warm, soft hues like mustard yellow symbolize hope and ambition, while Sebastian's (another character in the movie) cooler tones reflect his passion for jazz and internal conflict. The vibrant color contrasts, especially in scenes like the opening traffic dance, amplify the dreamlike, bittersweet quality of the film, highlighting its themes of love, dreams, and sacrifice.

In the dance sequence, titled "A Lovely Night" in the same movie, color plays a key role in expressing emotions and character dynamics. The warm orange sunset lighting brings forth romance and nostalgia, enhancing the dreamlike quality of the frame. The contrasting colors of their costumes reflect their differing personalities—Mia's lively spirit and Sebastian's grounded nature. While the urban background highlights their connection. Overall, the colors create a visual metaphor for the joyful yet complex relationship between the characters.

While music itself is not inherently visual, color is often linked with sound through album covers, music videos, and stage performances. For instance, the bold red cover of a rock album

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can convey energy and rebellion, while soft pastels might represent sweetness or nostalgia in pop music.

Music's emotional impact is often described using color metaphors. For example, major keys are linked to bright colors like yellow, representing happiness, while minor keys are associated with cooler tones like blue, evoking sadness or mystery. Fast tempos might suggest vivid colors like red, whereas slow tempos bring muted colors like grey. Musically, instruments and chords have distinct *colors* in terms of sound, on one hand, violins might be described as golden, and on the other, diminished chords as dark.

Additionally, music videos and live performances often use lighting and color schemes to enhance the song's emotional tone. Overall, color deepens the emotional and aesthetic experience of music.

Colors are an essential element of branding and design that can evoke powerful emotional responses, enhance brand recognition, and guide consumer decisions. When choosing colors for a brand, it's important to understand their psychological impact, cultural significance, and how they can be harmonized to support the overall message of the brand.

Each color carries psychological meanings, one may have observed that red evokes excitement, blue signifies trust, yellow conveys optimism, and green symbolizes nature and health. Consistency in color usage is key for brand recognition. Think about major brands like Coca-Cola, whose consistent use of red makes them instantly recognizable across the globe, or Nike, whose black-and-white palette along with the iconic 'swoosh' logo reflects power, simplicity, and elegance, or even Starbucks, whose green color represents freshness, sustainability, and relaxation, which aligns with the brand's focus on high-quality coffee and a comfortable atmosphere.

Colors can also be influenced by trends that shift over time, and brands may adopt trendy colors to seem current and relevant. For example, Pantone's 'Color of the Year' often influences brand designs and marketing campaigns.

In essence, colors are far more than just visual stimuli. They are an integral part of our emotional and psychological landscape- from the way we see them to how they influence our behavior and emotions, colors play a powerful role in shaping our experiences. Whether in movies, music, advertising, branding, or the world around us, they provide an additional layer of meaning, enhancing our understanding and connection to the world.

Article

Machine Learning is just Statistical Mechanics with Better Marketing

by Vaibhav Kalvakota

Introduction

Both machine learning and statistical mechanics work with the complexity of high-dimensional spaces[1], emergent properties, and stochastic dynamics. Now of course, statistical mechanics studies the macroscopic behavior of systems composed of many interacting components using results from probability and statistics to bridge the gap between microscopic details and observable phenomena. In fact, it does this *so well* that most of the intuition to the workings of the macroscopic world are built on it from the fine-grained details.

Machine learning uses probabilistic tools to learn patterns in large datasets, optimizing over parameter spaces to make predictions or discover underlying structures, to carry out a particular task. There are many aspects of statistical mechanics, such as Boltzmann distributions, free energy minimization, and stochastic processes that lie deeply within the tools and algorithms that drive ML, including stochastic gradient descent, regularization, and probabilistic modeling.

(There are actually many other overlaps and applications of pure mathematics in machine learning as well, which we will not discuss.)

Boltzmann

One of the most fundamental correlations between the two subjects – of machine learning and statistical mechanics – is the obvious use of a number of information theoretic components in machine learning. When we seek to define a loss function $J(\theta)$ (where θ are the parameters of the model), we usually define it to be something like the *cross entropy loss function*,

$$J_{CE}(P,\ Q,\ \theta) = -\mathbb{E}_P\left(\log Q\right), \tag{II.1}$$

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[1] By this, I want you to imagine a large parameter space and you want to find the optimal configuration in machine learning.

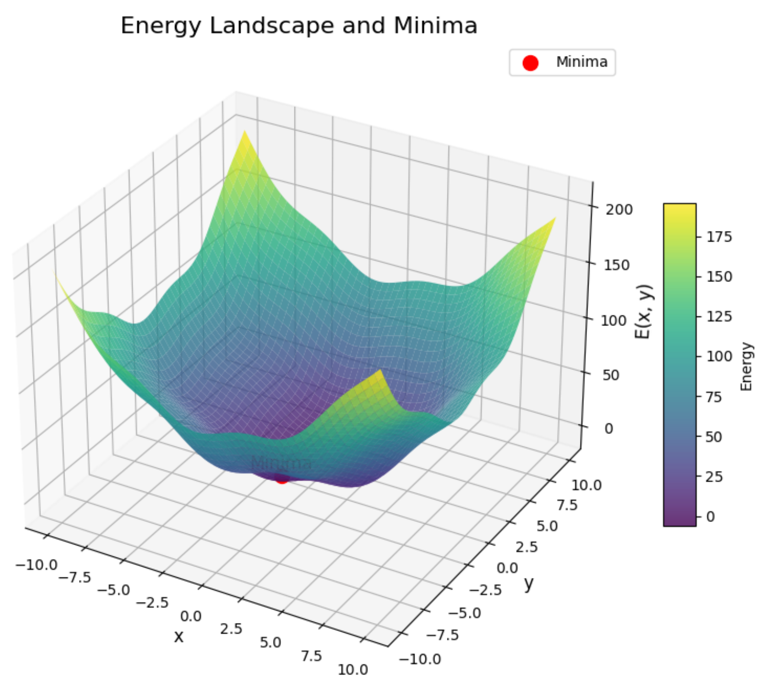

Fig: The minima is the little red dot in the landscape of different loss configurations. By Vaibhav Kalvakota

where P is the *true* distribution and Q is the *selected* probability distribution. $\mathbb{E}$ denotes the expectation value and this just becomes the usual form of $-P\log Q$. A loss function measures how "off" the model is from the actual probability distribution. Yes, yes, there's some measure theory stuff that happens when talking about this quantity rigorously. This is essentially measuring the Kullback-Leiblar divergence, but at the heart of it, all you need to worry about is that it tells you the "loss" of an observed distribution. Now what these loss functions also tell you is *how bad the other choices are*.

Imagine a huge parameter space. The problem in machine learning is to find the optimal configuration through this parameter space so that you have the least loss – best fit for the predictions – in the model. In the context of supervised learning[2], this is simply just making better predictions, and might involve some more tweaking with regularization/dropouts/etc.

[2]That is, when you are providing the outputs for the model to train on instead of just trying to find features in the inputs.

Figure-1 shows you the minima of the loss functions: the optimal state, so to speak, and this is *really hard* both computationally as well as theoretically sometimes, to reach. For most of the heavy machine learning models we use, the choice is that of *stochastic gradient descent* – a fancy way of using the extremum derivative rule for arbitrary "learning steps". This brings down the computational costs by a significant margin over normal gradient flow where you have to estimate with smaller strides. The price of using SGD over batch gradient descent is that you never really converge onto the minima; you just get really really close to it. As physicists, we obviously never complain about doing stuff like this.

In generative machine learning, more specifically a branch called ICA – **Independent Components Analysis**, you end up with a heavy reliance on *probabilistic models*. Simply put, given a probability distribution, you want to find the loss function minima as usual, but the way you do so isn't by wildly sampling[3] over the parameter space or training with more and more data; you instead view a *Boltzmannian approach* to machine learning. The slogan here is: *loss functions are no longer just minimas, they are energy functions, and you seek to minimize the energy.*

In a Boltzmann distribution, for some input x, you have an associated energy $E(x)$ and a probability given by

$$p(x) = \frac{\exp\left(-\beta E\right)}{Z} \ , \tag{11.2}$$

where Z is a normalization term. Of course, the physics people must have said *who are you kidding Vai that is a partition function, just call it that*, and yes it is a partition function[4] but bear with me for a second, will you?

The objective here is to minimize the energy loss function $E(x)$, but not with ordinary gradient descent. Instead, we want to use a *flow* $\mathcal{F}$ so that the action of $\mathcal{F}$ on the energy space[5] $\mathbf{E}$ is to make higher values of the energy function larger and the lower values smaller. So, you no longer just minimize by walking along the function, you stretch it out, by something like $\dot{x}(t) = -\nabla E(x)$, something like an inverse curvature flow.

This is something that has been done for quite a while in machine learning now, and Yann LeCun[6] has a very good paper on this. In working with energy loss functions instead of ordinary loss functions, we have to restrict to certain kinds of loss functions, but the most used loss function, the cross-entropy loss (11.2) is inherently a natural energy function.

[3]By this I mean taking steps in the parameter space to minimize the loss function.

[4]This field of research is called *canonical ensemble learning* where we seek to define partition functions and minimize the energy function.

[5]That is, a parametric space w.r.t the energy function.

[6]https://yann.lecun.com/exdb/publis/pdf/lecun-06.pdf

Attention Is All You Have

A natural sort of continuation from the above discussion is to talk about the partition function Z that I didn't want to specifically talk about. I still don't, because there are subtleties around actually computing it. Spoiler: you don't really consider it a "computable" quantity, but that is fine.

When you work with Transformer neural networks, you usually work with the softmax function by taking in the raw logits z_i and producing a weighted attention weight p_i:

$$\text{softmax}(z) = \frac{\exp z_i}{\sum \exp z_i} \ . \tag{II.3}$$

Look carefully at the denominator term. It is technically just a partition function, and this would generate "moments" akin to correlators like $\langle x_1,\ x_2,\ x_3 \ldots x_n \rangle$.

The first moment is the Helmholtz free energy $F = -\log Z$, which is a very interesting term. But there are more terms that also appear in working with these models, the most important of them being the *entropy* S. This is the usual Shannon entropy, but there are many useful things that come out of these two terms.

The entropy

$$S = -p_i \ \log p_i \tag{II.4}$$

calculates the uncertainty of the model. And LLMs are all about making that trade-off between very deterministic responses and overly diverse responses. This is a very simple task indeed; for even something as well trained as GPT-4o mini, you can end up with responses that will be too deterministic. The most natural way an LLM generates an output is by *greedy sampling*, where it simply picks the tokens with the highest post-softmax'd weights. However, the issue with this is that for out-of-distribution scenarios, the responses will be – trash. So models typically use *temperature* to make more diversified generations. There is a trade-off between strictly deterministic and highly diverse outputs that models require, and it was suggested early this year that there be dynamic temperature sampling using entropy. Which is really *really* interesting, and more recently, @xjdr started Entropix, which is (last I remember) a Llama 3.1 model with entropy sampling. I am, in fact, working on related things with entropy sampling for attention sparsity that could potentially make the complexity order of these models less than $O(n^2)$ or $O(n\sqrt{n})$ as is usually expected from adaptive sparsity. You could, alternatively, use the free energy, which technically captures more information than the entropy. In fact, sampling with free energy would work on two sides: one, it would focus on the lower energy functions subspace of $\mathbf{E}$ and could potentially sample a larger subset, and two, it would update the usual uncertainty metrics like entropy and variance-entropy. However, I am unaware of any models that use this yet.

Let me illustrate two machine-learning-in-physics topics that I think are really interesting. This is of course going to be somewhat more technical than what I talked above. [7]

Calabi-Yau Machine Learning

This is one of the most ambitious projects I have seen in quite some time in theoretical physics that has numerical calculations. Calabi-Yau manifolds are Ricci-flat objects[8] in string theory that have many applications in dimensional compactifications of extra dimensions.

Without going into too much technicalities, due to a high level of sophistication as well as my own lack of competence to phrase the technicalities in a readable format, the basic idea is just that Calabi-Yau manifolds are closed Kahler manifolds (these are complex manifolds with a hermitian metric). More specifically, a Calabi-Yau manifold is a compact Kahler manifold so that equivalently, (1) the first Chern class[9] vanishes, and (2) there exists a g that is Ricci flat. Another way of stating this is that there is a non-vanishing holomorphic n-form or has holonomy[10] in the special unitary group SU(n). These manifolds generally appear in string theory when working with dimensional compactifications, as in taking a description of a 10D $N = 1$ SUSY theory with a low-energy limit on a $D = 4$ manifold, with the remaining 6 dimensions reduced onto a Calabi-Yau manifold. There are certain topological invariants called *Hodge numbers*, which for some level of vague intuition, are related to the Euler characteristic of the manifold.

At a high level, we basically just want to be able to calculate these Hodge numbers. You can just think of this process as taking a list of pre-calculated complete-intersection Calabi-Yau manifolds data with the Hodge numbers and train a neural network using supervised learning and predict Hodge numbers simply, by taking a huge corpus of these Calabi-Yau data, it is possible to train a neural network that predicts the Hodge numbers very effectively. There are other Calabi-Yau calculations where neural networks help, see for instance a recent paper by Manki et al[11]. See also this paper[12] which has some excellent discussions on the string theory arena of machine learning.

Discrete Theory Space 101

The above discussion of CY manifold machine learning was straightforward to speculate on. However, in many cases, there are no *discrete* theory spaces for a particular constraint problem.

[7] To be fair, all I did was talk about energy based models.

[8] That is, the contracted Ricci tensor is zero, while the Riemann tensor need not be zero.

[9] https://en.wikipedia.org/wiki/Chern Class

[10] See for instance, https://ncatlab.org/nlab/show/holonomy

[11] Cristofero S. Fraser-Taliente, Thomas R. Harvey, and Manki Kim. 2024. Not So Flat Metrics. 11, https://arxiv.org/abs/2411.00962

[12] Andrei Constantin. 2022. Intelligent Explo- rations of the String Theory Landscape

My example of this would be the Wheeler-DeWitt equation,

$$H\Psi[g, \Phi] = 0 \ , \tag{II.5}$$

where g is the metric and Φ are the matter fields on the manifold (M, g). States of this constraint are hard to solve unless you impose specific conditions such as asymptotic bulk limits, restriction to isometry groups, etc. and compose the Hilbert space of perturbative canonical quantum gravity when solved around perturbations. One could ask if there is a machine learning optimization task that could help us solve constraint (II.5). This is a technical problem but just consider the following. Take the decomposition of WDW states $\Psi[g, \Phi]$ into two "branches", Z^+ and Z^-. This happens because the Hamiltonian constraint is quadratic in nature, and usually there is one dominant branch. In any case, the collection of Z generate the so-called "theory space" and have a universal counterterm $S[g, \Phi]$ from holographic renormalization that applies to the entire theory space. This is good.

However, if you want to model neural networks that predict these counterterms, unlike the prediction of Hodge numbers, you end up with terms that are not "discrete", in the sense that numerically, it does not make sense to have a collection of universal counterterms that define a particular theory space simply because there aren't a discrete subset of these to begin with. You could seek to define other things that could be more numerically discrete, so that for specific cases you have specific values and then try to predict the values for the term in other cases. This goes on to just illustrate a level of obstruction for what can be computed and what cannot be, even if fundamentally they are just purely numeric coefficients.

In Phenomenology

Often in physics, we have to calculate physical couplings for theories. These are quantities that tell us a lot about the theory and the concretely observable properties of the theory as well. In particle collider experiments such as at the LHC (CMS or ATLAS), by computing the phenomenology of particle collisions, you can gain a lot of insight into the interaction between, say, partons. As a way of illustrating the role of machine learning in such phenomenological calculations, when working with QCD, it is very important to work with *parton distribution functions*, which are measured in partonic interaction experiments and give information about the cross-sections and interactions. The usual way of calculating or checking these distribution functions are either too complex or too time-taking, due to which large-scale computations become less feasible. In this paper[13], the collaborators trained a neural network to calculate the log-likelihood χ^2 from parton experiments at LHC. In fact, machine learning finds several roles in hadronic physics and pheno/experiment research, such as in working with meson production, heavy-ion collisions, and even beyond standard model particle interactions.

[13] DianYu Liu, ChuanLe Sun, and Jun Gao. 2022. Machine learning of log-likelihood functions in global analysis of parton distributions. JHEP, 08:088

Theory vs Experiment

Here I would like to draw a comparison that I think is very important. In high energy physics theory, there is an inherent distinction between theory and experiment. When I say that there is holography in an evaporating black hole spacetime, I typically mean that there are some concrete observables, but these aren't observables that you can actually calculate in real life as a part of an experiment.

For that matter, one of the last theory-meets-experiment timelines we had was closed around the time the CP violation was observed, because it was an observable phenomena. We do not have a way to calculate the amount of radiation collected from an evaporating black hole in anti-de Sitter space or the entropy of an island in AdS/CFT. This is the case with a lot of hep-th. The case with machine learning is the polar opposite; there are significantly good resources to actually check a theory. This has been known from the time we had RNNs and LSTMs, which still hold up nearly as well as some of the smaller-end base model Transformer architectures like BERT. In such cases, the line between theory and "experiment" must not be drawn, and the mathematical (often referred to as "pedantic") aspects that comprise machine learning should not be discarded. Since, after all, machine learning is essentially statistical mechanics with better marketing.

However, there is an important distinction between physics and the *applied* physics aspects of machine learning. By this, I mean that there are many things like Ising models, diffusion models, Langevin dynamics, phase transitions, etc. that are used in machine learning that arise from statistical mechanics. However, this does not count as doing *physics*. I say this because the 2024 Nobel Prize in physics was controversial and many (hep-th) academics questioned the principles on which this was given. However, if there is stringent applied physics being used in things like quantum computing or machine learning, this lies within the domain of "physics" in general.

Conclusion

In summary, there are a lot of interesting stuff to work on in machine learning and physics, and the overlaps between the two fields[14], and you should go check out the arXivs for hep-th and cs-LG (or stat-ML and cs-AI). And the so-called "this part of Twitter" (tpot) will illustrate the necessity of this article.

[14] There's also stuff with algebraic geometry in *singular learning theory*, which has some really exciting mathematical prospects. See *Sumio Watanabe. 2009. Algebraic Geometry and Statistical Learning Theory. Cambridge Monographs on ACM. Cambridge Press.*

Article

The Enigma of a Dream

by Utkarsh Kashyap

"Alright, Shridhar, tell me honestly – what really happens to those who die? I mean, I've heard people talk about spirits and all… Is any of that true?"

Shridhar replied, "Oh! Come on, Samir! How did you fall for all that nonsense? It's all rubbish."

Was Shridhar really that certain? So certain that he didn't think twice before using the word 'rubbish'? Maybe he saw himself as just another victim of the so-called human nature- the same nature that only becomes evident when someone tries to belittle or demean the other without any solid fact or logic, without even bothering to understand the impact such words might have on the other's mind and life; how each spoken word has the power to bring profound change- not only in that person's life but also in the lives of generations to come. Yet, he believes that, just as others say whatever they please without any thought and he fails to see through their web of illusions- his own words too will be accepted thoughtlessly.

I don't know about others, but I'm certainly not one of those people. If I were to become like them, what difference would there be between me and someone dead? Perhaps he knew his words wouldn't have much impact, so he felt free to say whatever he wanted.

I ended up thinking more about Shridhar than listening to what he was actually saying.

"You're saying this only because you've never had an encounter with a spirit," Nirmal interrupted, disagreeing with Shridhar. But I don't think anyone had asked him anything. Maybe he realised that, and suddenly turned to comfort Samir, saying- "But don't worry. Just take care of him for these thirteen days well enough, and then let him be free."

What? Let him be free? Had Samir somehow imprisoned his father's spirit? He wasn't such a terrible person and even if he had mistakenly been so cruel, he now wishes to free his father's spirit, and he can do so right this moment. What sense does it make to torment him for thirteen days before setting him free? (If I seem as naive as I appear here, then the one trying to explain these things to me should be equally convincing. And if that doesn't happen, then perhaps either I am not as naive as I appear, or no one truly has any grasp of these airy, baseless beliefs.)

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My facetious thoughts seemed to travel from my mind to my face, dissolving into the atmosphere, and Samir could clearly see this little journey unfolding in my expressions. Anyway, what happened next was not what I feared, but what was simply obvious.

"Tarun, tell me, what's your take on all this?" Samir asked me while deforming the cot. Shridhar and Nirmal's answers didn't satisfy him. Was he expecting something from me? I would have to break that expectation. "I don't know," I replied.

Was that all? Yes, that was it.

For me, it seemed like the only straightforward truth.

Just yesterday, Samir was describing the mysterious things happening at his home with great seriousness, and the listeners were nodding along. Shridhar was also one of them. So, what's the point of raising such questions today when he is about to provide answers to them tonight only?

I've heard these legends, in fact, the entire village has heard them. What's the point of repeating them over and over again, or what's the pleasure in opposing them, when neither side really knows the truth?

My response was brief, but it was honest and sufficient. Yet, nobody seemed particularly pleased with it, and gradually, I was pushed out of their conversation (or rather, ignored). The three of them continued crafting stories about life after death for Samir's father, with Shridhar firmly seated, bringing his so-called science into the discussion and making significant contributions. After listening for a while, I realised which treasure chest they were pulling their stories from. After all, what fresh perspectives could these boys in their early twenties truly bring?

Then, I started gazing at the setting sun. Ah, this setting sun! It feels deeply solemn and affectionate, as if at the moment of its departure, finding only me before it, it conveys the sorrow of separation hidden in its heart to mine in such a way that no one else could sense this parting. And as it fades silently, it seems to promise an experience yet to come—one with the power to embrace my entire night in its hold.

Suddenly, the aroma of some delicacy caught my attention, drawing me towards the house three doors down. Ah! Amma had made Poha today. Anyway, no one needs to call me to eat; I appear in the kitchen on my own for such things. She knew this, and so my meal was already served. Enjoying my food, I asked, "Amma, what happens after death?"

She replied, "How would I know, child? I'm still alive. Ask someone who's dead, someone who knows what dying feels like, because that's beyond my experience."

Amma's answer seemed better than mine.

She then asked me, "Why are you suddenly so eager to uncover this mystery today?" I narrated the whole incident to her.

"Yes, Amma, but you're right," I said. She then replied with a playful tone, "But you're wrong. If you didn't blindly believe what Shridhar and Nirmal said, then why believe my words? I could be wrong too."

What did she mean? In other words, she meant to figure it out for yourself. You're capable enough, aren't you?

As always, she left me speechless again. Amma is so wise – where does she get all this wisdom? I mumbled to myself as I headed to the rooftop and began gazing at the moon. But a harsh voice interrupted this tranquil moment saying something like this- "And then, suddenly, someone knocked on the door and called out to me saying, 'Let me in, son. I need my evening fix.' I was terrified. Father is no more, then who was this impersonator?"

Hearing the loud proclamations being shouted into the darkness of the night, I managed to piece together a few things-

First, someone was declaring their father to be both dead and an impersonator, tarnishing his name in front of the villagers. Second, this person's father seems to only come home suddenly at night when everyone is asleep, solely to collect his "fix".

Where he spends the rest of his day, no one knows. And if this "fix" is so important, why not keep it with him? What's the point of leaving it behind at home?

My suspicion turned into certainty, and I realized exactly whose father was being spoken about.

Laughingly, I lay down comfortably on the cold cot. Coincidentally, lying in this relaxed position allowed me to see the moon perfectly without straining my neck muscles. Tell me Moon, how can I find out about life after death? But Amma was right. I haven't even seen death, so how could I possibly know? And at the moment, I have no particular intention of dying. Perhaps this is a topic best pondered after I've experienced it firsthand.

Don't you think that since I'm alive, I should be thinking about life instead? But what is there to ponder about life? I already know about it, and there's no real curiosity left. So, unable to find any truly interesting topic to think about, I began to try to sleep. Just then, I stumbled upon something strange yet intriguing: that I can't explore things that happen after death, and I have no particular interest in things before death, so why not focus on that boundary where both meet , that is, "death"?

But since I have no experience with death, I might as well fall asleep and venture into the world of dreams.

Under the dense darkness of the winter sky, Nandini was enjoying the fodder in the trough before her. Piercing through the dense darkness, the light of a lone earthen lamp was performing the noble task of offering us both a faint warmth. But to me, it was providing something beyond this warmth— the tender yet profoundly divine face of Nandini as she grazed on her fodder. I was almost enchanted watching her and the sparkle in her eyes. She was the same as ever but today... As I watched her, no questions about life, death, or the afterlife disturbed my mind. I was so absorbed, so immersed, that these questions seemed meaningless as if they had ceased to exist.

Once her hunger was satisfied, she slowly lifted her head up. It seemed as though her physical hunger was gone, but there was another longing left to satisfy. And for some reason, I too, felt a strange restlessness from within – a restlessness I'd never experienced before, one that could only be calmed by her presence. It was as if my gaze had mesmerised her too. We both continued to stare at each other without reason, just silently, endlessly... But why? I didn't know.

Just as a plant slowly begins to lose its sap when being cut, it felt to me as if someone was cutting me from deep within, with its sap collecting in my eyes as tears, as if the eyes had forgotten how to blink. Tears had indeed turned into droplets, but even these droplets seemed to have forgotten how to fall. My entire body, every thought, every sense, the whole atmosphere— everything had become completely still. I wasn't sad, but I wasn't happy either. Had someone taken control over me? But I don't believe in any of this. Only mentally weak people fall prey to such things, don't they? but I wasn't like that. I don't believe in anything without reason, so what was this in reality?

I gently touched Nandini's two white cheeks and pressed my forehead against hers, wanting the stream of tears to flow. Doing so felt as essential to me as breathing was to my body. Maybe I had gone mad; surely, this was madness, but why was this madness so blissful?

Then I did just that, and Nandini also soaked her face in her own stream of tears. I was neither sad nor happy; it was as if I had disappeared; despite knowing about this state, I was unaware of it. Now Nandini had become life itself, but I used to embrace her every day. So why this extraordinary experience today?

Slowly, a strange bright light enveloped the entire atmosphere—it was the same ray of the sun that dared to break such a beautiful dream.

What? Was it all just a dream?

How could the sun do this? Just yesterday, it seemed so full of love, so close to my heart, and it was the same sun that gave me a glimpse of that dream. And now, it has committed such a vile act by breaking it. Look at it now, so cheerful after rising, as if it has done me a great favor. Oh! What has it done? I can't bear to think about it even for a moment. I was very sad. Very, very disturbed. I had never been so disappointed. There wasn't even a cow named Nandini in our house; it was all a dream. Sadness was clearly visible in my eyes.

It was as if the tears wanted to step outside, just to gaze at those sorrowful eyes themselves.

I neither felt hunger nor thirst.

Perhaps I truly was going mad. I tried to sleep again, hoping to revisit that same dream, but the trick didn't work. For the first time, something irrational kept me entangled for days. Even today, when I recall that dream, I am filled with emotions and tears, lost in silence and fading into the strangeness of that incident.

Fig: Generated using AI. Utkarsh Kashyap

Resources Palette

Books

- The Men of Mathematics by Eric Temple Bell
  The book is about many mathematicians and contains short biographies of their lives and their mathematics.
- What is Art by Leo Tolstoy
- Lilavati's Daughters by Ramakrishna Ramaswamy and Rohini Godbole
  A collection of nearly 100 biographical essays on Indian Women Scientists.

Web Resources

- A page maintained by Yuji Tachikawa about unofficial mirrors of every Strings conference which also contains talk's slides
  https://member.ipmu.jp/yuji.tachikawa/stringsmirrors/
- Bhavanā Magazine - The Mathematics Magazine
  https://bhavana.org.in/
- Lecture notes by David Tong on various subject of theoretical physics
  http://www.damtp.cam.ac.uk/user/tong/teaching.html
- Open Problems in Operator Algebra, maintained by Jesse Peterson
  https://math.vanderbilt.edu/peters10/problems.html
- Physics Latam Seminars Archive
  https://www.physicslatam.com/seminar
- Oral History Interviews by AIP
  https://www.aip.org/history-programs/niels-bohr-library/oral-histories
- Expository Papers by Keith Conrad on various subjects in mathematics
  https://kconrad.math.uconn.edu/blurbs/
- Introductory Literature Suggestions on Fields, Strings, Symmetries, and Integrable Systems, maintained by Alexei Vladimirov
  http://theor.jinr.ru/ālvladim/lit.html
- The Unreasonable Effectiveness of Mathematics by Eugene Wigner.
  Available at https://www.maths.ed.ac.uk/ v1ranick/papers/wigner.pdf

Upcoming Conferences and Programs

- Upcoming conferences in algebraic geometry maintained by Ravi Vakil
  https://virtualmath1.stanford.edu/ vakil/conferences.html

- **STRINGS 2025** (Jan 6, 2025 - Jan 10, 2025) at NYU Abu Dhabi.
  Notable subjects include de Sitter, operator algebras, information theory, and string theory.
  https://nyuad.nyu.edu/en/academics/divisions/science/strings-conference-2025-abu-dhabi.html

- **What is Particle Theory?** (Jan 6, 2025 - Apr 11, 2025) at KITP, Santa Barbara.
  A lengthy program at KITP to discuss the topics in fundamental physics.
  https://www.kitp.ucsb.edu/activities/particles25

- **A Hundred Years of Quantum Mechanics** (Jan 13, 2025 - Jan 17, 2025) at ICTS, Bangalore.
  https://icts.res.in/discussion-meeting/QM100

- **Recent Advances in Mathematics and Related Areas** (Jan 14, 2025 - Jan 19, 2025) at SRM University, Andra Pradesh.
  https://srmap.edu.in/recent-advances-in-maths/

- **Workshop on Symmetries and Gravity** (Jan 21, 2025 - Jan 25, 2025) at CMSA, Harvard.
  https://cmsa.fas.harvard.edu/event/symmetries/

- **Positive Geometry in Scattering Amplitudes and Cosmological Correlators** (Feb 10, 2025 - Feb 21, 2025) at ICTS, Bangalore.
  https://www.icts.res.in/program/PosG

- **Emmy Noether Workshop: Quantum Space Time** (Mar 10, 2025 - Mar 14, 2025) at Perimeter Institute, Waterloo.
  https://events.perimeterinstitute.ca/event/928/

- **37th Automorphic Forms Workshop** (Apr 30, 2025 - May 04, 2025) at University of North Texas, USA.
  http://automorphicformsworkshop.org/index.html

- **Automorphic Forms and the Bloch–Kato Conjecture** (May 19, 2025 - May 30, 2025) at ICTS, Bangalore.
  https://www.icts.res.in/program/afbk

- **Threads in a Theory Tapestry, TASI 2025** (June 2, 2025 - June 27, 2025) at University of Colorado, Boulder.

https://www.colorado.edu/physics/events/summer-intensive-programs/theoretical-advanced-study-institute-elementary-particle-physics-current

- **String Math 2025** (June 23, 2025 - June 28, 2025) at BIMSA, Beijing, China.
  The intersection of string theory and mathematics.
  https://bimsa.net/conference/String-Math/
- **Les Houches on Dark Universe** (July 7, 2025 to Aug 1, 2025) at Ecole de Physique des Houches.
  https://indico.iap.fr/event/25/page/16-lecture-program
- **CIMPA School on Automorphic L-functions** (June 30, 2025 - July 11, 2025) at IIT Ropar.
  https://www.mathconf.org/alfiitr2024
- **Gravitational Waves from Theory to Observation, PITP 2025** (July 14, 2025 - July 25, 2025) at IAS, Princeton.
  https://www.ias.edu/pitp